1 Introduction to Physics

Much of calculus was developed to study physics. While a course in physics is not required for calculus, there are a few fundamental concepts from physics that we will use over and over again. This is a summary of those concepts.

Most of us are familiar with driving a car. The speedometer measures the **speed** at which you're traveling, while the odometer measures the **distance** you've traveled.

Example 1

Let's suppose you take a two-hour drive, and you drive at a constant rate of 30 km/hr. Below is a graph of your starting position, at $x = 0$. What is your ending position?

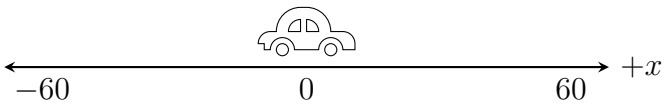


Figure 1: Starting position.

If you drove east for two hours (we'll describe going in the positive direction as going east, and going in the negative direction as going west), you'd be 60 km east of where you began.

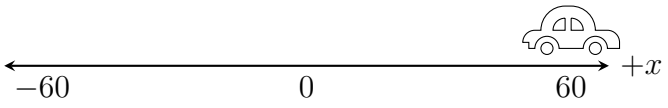


Figure 2: Ending position A.

But if you drove west for two hours, you'd be 60 km west of where you began.

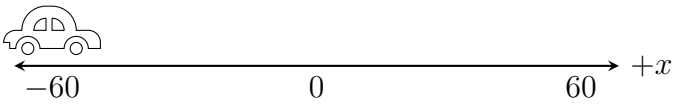


Figure 3: Ending position B.

But maybe you drove east for one hour, and then turned around and drove west for an hour. Then you'd be right back where you started.

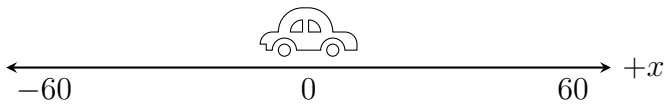


Figure 4: Ending position C.

Since you're driving at a constant speed, your speed graph would look like this.

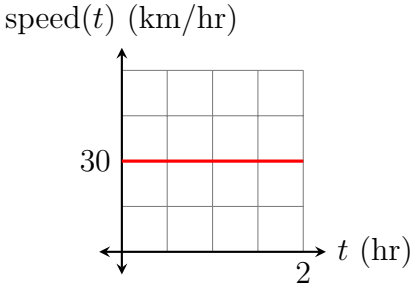


Figure 5: Speed graph.

The main issue is this: you can't know your ending position by looking at the speed graph. There is not enough information. That is why the concept of **velocity** is so important in science. Essentially, velocity is speed *and* direction. When you're driving east, your speed and velocity are both 30 km/hr. But when you're driving west, your speed is 30 km/hr, but your velocity is −30 km/hr.

Let's look at the velocity graphs corresponding to the three different ending positions, A, B, and C. For position A, speed and velocity are the same: 30 km/hr for two hours. The letter "v" is used in physics to represent velocity.

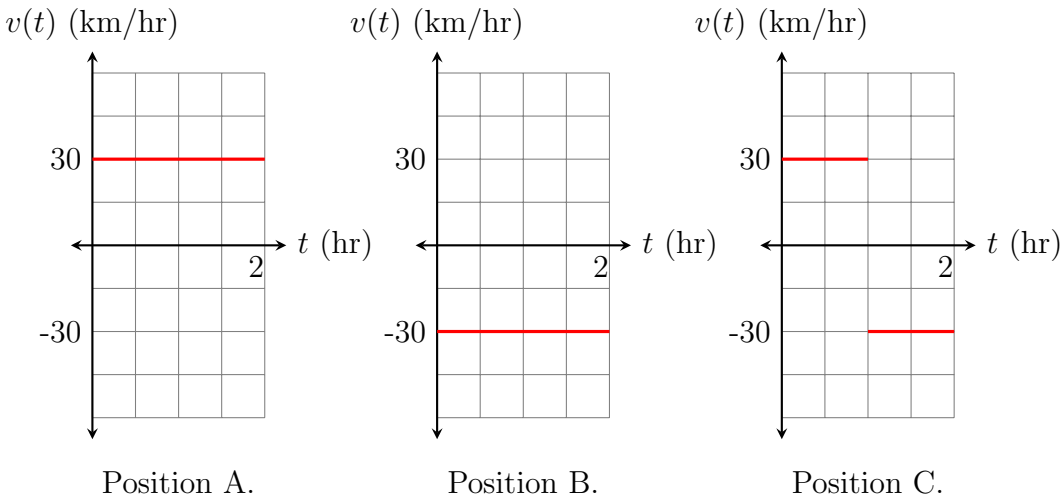


Figure 6: Velocity graphs for different ending positions.

Now for the ending position B, you drove west for two hours, and so your velocity is -30 km/hr for two hours. For the ending position C, you drove east for one hour (30 km/hr) and west for the next hour (-30 km/hr). So the velocity curve jumps down to -30 after one hour.

The important point is this: if I gave you one of the *velocity* graphs in Figure 6, you could tell me *exactly* where I ended up. But all three ending positions have the *same* speed graph in Figure 5. So in physics and science, "velocity" is a much more useful concept than "speed."

Example 2

Let's continue with the same three scenarios, but this time, we'll focus on the distance traveled. Since distance equals rate times time and you're driving at a constant speed, you've traveled $30 \times t$ km in t hours. So the distance traveled up to time t is represented by the graph below.

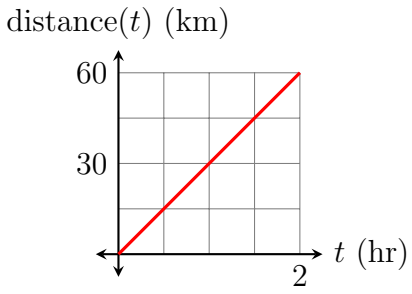


Figure 7: distance(t) $= 30t$

So this graph tells you that you've driven a total of 60 km, but there is no way to know *where* you ended up. Where you end up relative to where you began is called **displacement** in physics. Similar to how velocity is speed with a direction, displacement is *distance* with a direction. So your displacement in Ending position A is 60 km, but your displacement in Ending position B is -60 km, since you ended up 60 km west of where you started. Your displacement in Ending position C is 0 km, since you're back where you started.

Let's look at displacement graphs corresponding to the three different ending positions. In physics, displacement is often represented by the letter "s."

For Ending position A, your velocity is always positive, so you're driving east and ending up 60 km from where you started. For Ending position B, your velocity is negative, and you end up 60 km west from where you started: a total displacement of -60 km. Notice that the displacement graph has a positive slope when your velocity is positive, and a negative slope when the velocity is negative. This is not a coincidence, but lies at the heart of how calculus works. We'll definitely revisit this concept later.

Now for Ending position C, you start driving east, so the displacement graph is sloping upward. But after an hour – since you turned around – the displacement graph begins

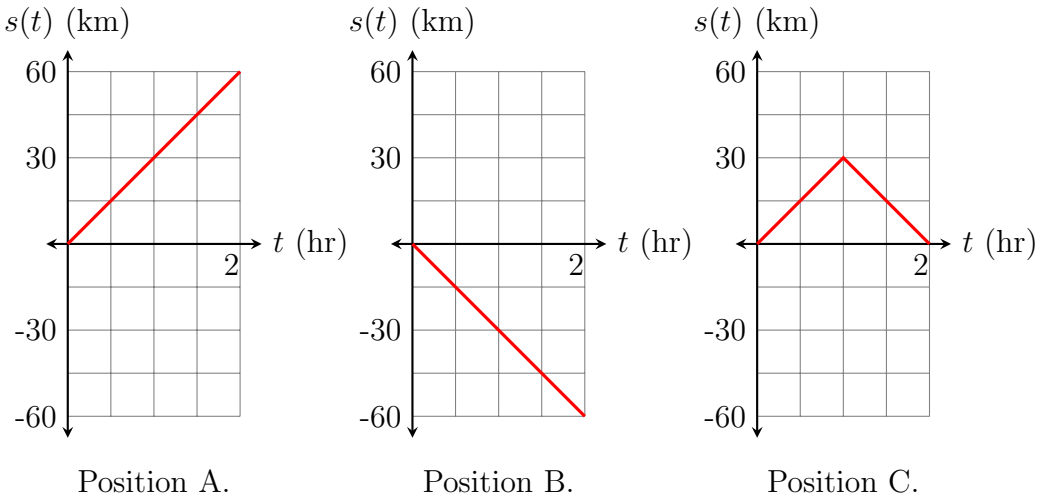


Figure 8: Displacement graphs for different ending positions.

sloping downward, so by the time two hours have gone by, your displacement is 0 km, since you ended where you started.

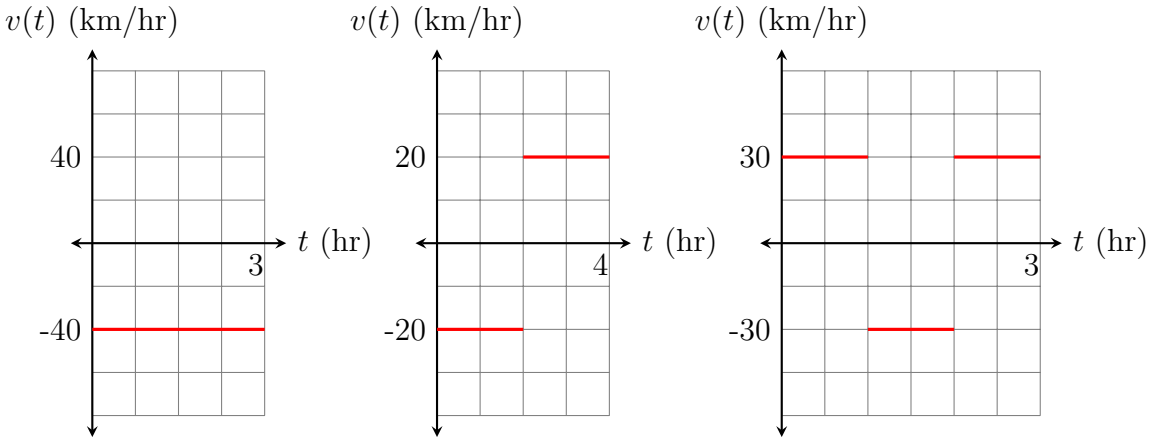
The important point is this: if I gave you one of the displacement graphs in this figure, you could tell me *exactly* what my trip looked like and where I ended up. But if I just gave you the distance graph as in Figure 7, the *only* thing you could tell me was that I drove 60 km. There would not be enough information to conclude any more about the nature of my trip.

Summary

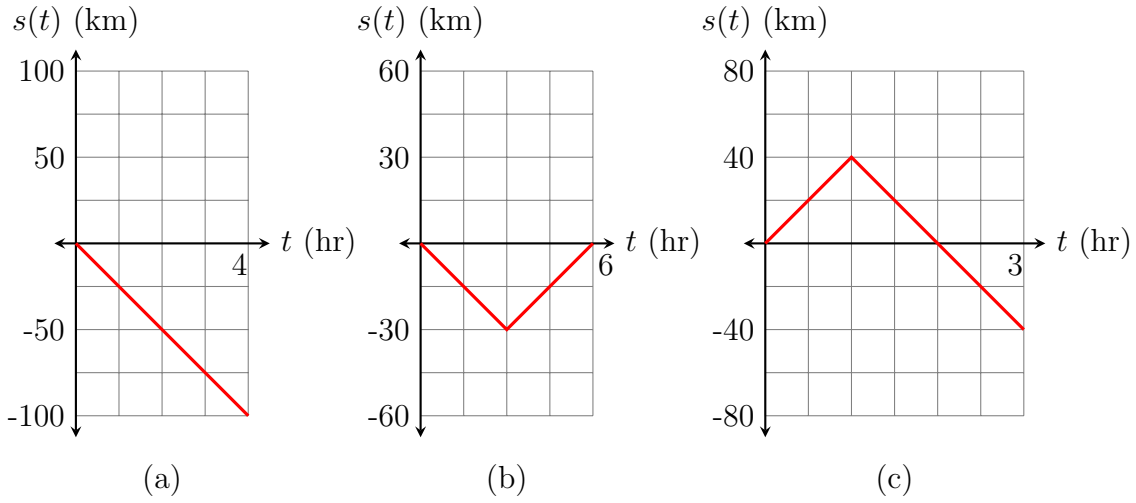
While speed and distance are concepts well-suited to day-to-day life, in the world of physics and science, they are not very precise. Graphs of speed and distance provide very little information about the nature of a journey. However, by introducing the concepts of velocity and displacement, we get an extremely accurate representation of what is actually going on. Essentially, calculus is an in-depth study of the relationship between velocity and displacement graphs.

Practice Problems

1. Each of the following is the velocity graph of a car trip. For each graph, (1) write a sentence explaining the trip in words, and (2) draw the corresponding displacement graph. Label your graphs carefully!

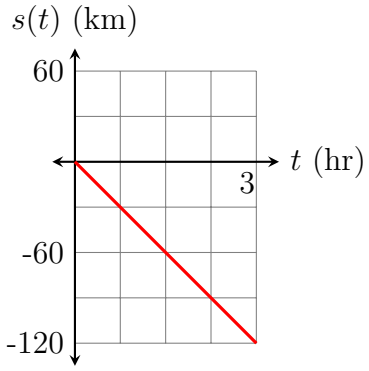


2. Each of the following is the displacement graph of a car trip. For each graph, (1) write a sentence explaining the trip in words, and (2) draw the corresponding velocity graph. Label your graphs carefully!

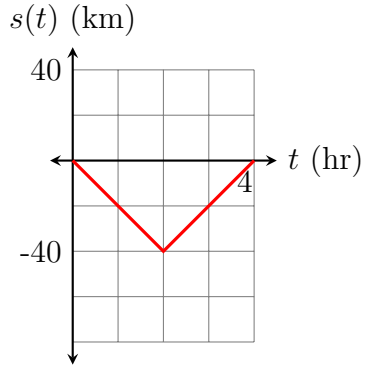


Solutions

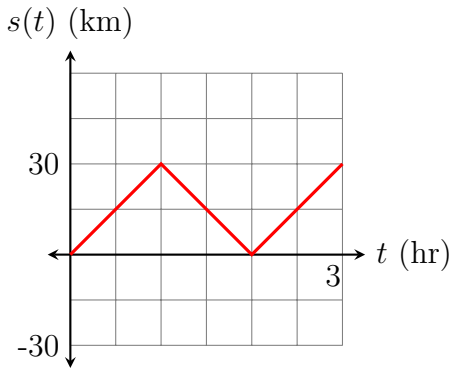
1. (a) You drove 40 km/hr west for three hours.



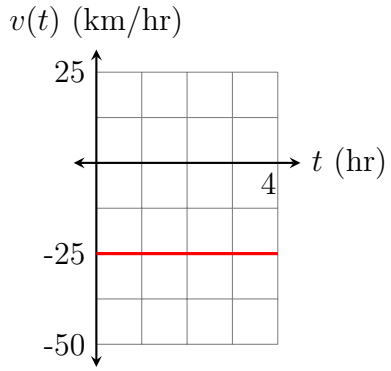
(b) You drove west at 20 km/hr for two hours, and then turned around and drove east at 20 km/hr for two hours.



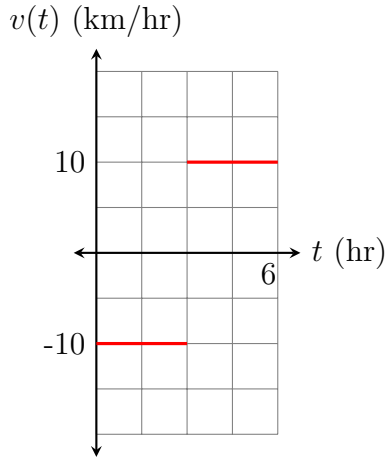
(c) You drove east at 30 km/hr for one hour, turned around and drove west at 30 km/hr for another hour, and then turned around and drove east for one hour at 30 km/hr.



2. (a) You drove 100 km west in four hours, and therefore drove at a velocity of -25 km/hr.



(b) You drove 30 km west in three hours – at a velocity of -10 km/hr, and then drove east 30 km in three hours – at a velocity of 10 km/hr.



(c) You drove 40 km east in one hour (40 km/hr), and then turned around and drove 80 km west in two hours (a velocity of -40 km/hr).