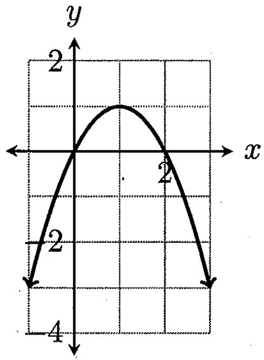
1. Let $f(x) = 2x - x^2$. Answer the following questions. The graph is shown below for reference only. All your answers must be justified *using calculus,* not just by looking at the graph.



+8

(a) Find the equation of the tangent line at $x = 3$.

$f'(x) = 2 - 2x$

$f'(3) = 2 - 2\cdot 3 = -4 \rightarrow$ slope

$f(3) = 2(3) - 3^2 = -3 \rightarrow$ point $= (3, -3)$

$y - (-3) = -4(x - 3)$

$y + 3 = -4x + 12$

$y = -4x + 9$

+3

(b) Where is the function increasing? Write your answer in interval notation.

$f'(x) > 0$

$2 - 2x > 0$

$2x < 2$

$x < 1 \rightarrow (-\infty, 1)$

+3

(c) Where is the function decreasing? Write your answer in interval notation.

$f'(x) < 0$

$2 - 2x < 0$

$2x > 2$

$x > 1 \rightarrow (1, \infty)$

Find the derivatives of the following functions.

+4 (a) $f(x) = x^3 + \frac{1}{x^3}$ $= x^3 + x^{-3}$

$f'(x) = 3x^2 - 3x^{-4}$

+6 (b) $f(x) = x^3 \cos(x)$ $f(x) = x^3$ $g(x) = \cos(x)$

$f'(x) = 3x^2$ $g'(x) = -\sin(x)$

$f(x)g'(x) + g(x)f'(x)$

$-x^3 \sin(x) + 3x^2 \cos(x)$

+6 (c) $f(x) = \frac{1+ax}{1-ax}$ $\leftarrow f(x)$ $\leftarrow g(x)$ $f'(x) = a$ $g'(x) = -a$

$$\frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2} = \frac{(1-ax)(a) - (1+ax)(-a)}{(1-ax)^2}$$

$$= \frac{a - a^2x + a + a^2x}{(1-ax)^2} = \frac{2a}{(1-ax)^2}$$

+2 Extra Credit: Recall that $\sec(x) = \frac{1}{\cos(x)}$. Use this to show that $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$.

$f(x) = 1$ $g(x) = \cos(x)$

$f'(x) = 0$ $g'(x) = -\sin(x)$

$$\frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2} = \frac{\cos(x) \cdot 0 - 1 \cdot (-\sin(x))}{\cos(x)^2} = \frac{\sin(x)}{\cos(x)^2}$$

$$= \frac{1}{\cos(x)} \cdot \frac{\sin(x)}{\cos(x)} = \sec(x)\tan(x)$$