SOLUTIONS
EXAM 3

1a. $h'(x) = 5^{4-3x} \ln 5 \, (-3) = -3 \ln 5 \cdot 5^{4-3x}$

b. $h'(x) = \frac{1}{(1-x^2)\ln 3} \cdot (-2x) = -\frac{2x}{(1-x^2)\ln 3}$

2. $y = e^{xy}$

$\frac{d}{dx} y = \frac{d}{dx} e^{xy}$

$\frac{dy}{dx} = e^{xy} \frac{d}{dx}(xy)$

$\frac{dy}{dx} = e^{xy}\left(x\frac{dy}{dx} + y\right)$

$\frac{dy}{dx} = xe^{xy}\frac{dy}{dx} + ye^{xy}$

$\frac{dy}{dx} - xe^{xy}\frac{dy}{dx} = ye^{xy}$

$(1 - xe^{xy})\frac{dy}{dx} = ye^{xy}$

$\frac{dy}{dx} = \frac{ye^{xy}}{1-xe^{xy}}$

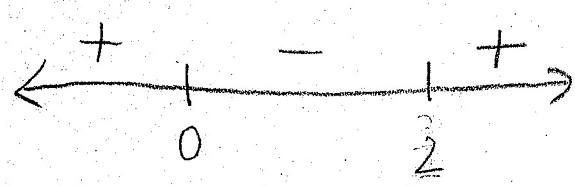
3a. $\lim_{x\to 0} \frac{x}{\sin x} \overset{LR}{=} \lim_{x \to 0} \frac{1}{\cos x} = 1$

d. $\lim_{x\to\infty} \frac{e^x}{x} \overset{LR}{=} \lim_{x\to\infty} \frac{e^x}{1}$ DNE $(+\infty)$

5. $\frac{1}{\sqrt{1-(1-x)^2}}(-1) = -\frac{1}{\sqrt{1-(1-2x+x^2)}} = -\frac{1}{\sqrt{2x-x^2}}$

6. $\lim_{x\to 0^-} \frac{2-3x}{x\sin x}$ is of the form $\frac{\infty}{0}$. $\frac{+}{+} \to$ DNE$(+\infty)$

7a. $3-2x=0$, $x = \frac{3}{2}$, $y = \frac{\frac{3}{2}-1}{\frac{27}{8}} = \frac{1}{2}\cdot\frac{8}{27} = \frac{4}{27}$

b. 

CU: $(-\infty, 0), (2, \infty)$

CD: $(0, 2)$

Please show as much work as you need. Keep in mind that if you skip several steps and get an incorrect answer, it will be very difficult to assign partial credit. Please take a moment to skim through the test first and start with the questions you feel most confident about. Your answers do NOT have to be in order on your paper.

1. (12) Find the derivatives of the following functions.

   (a) $h(x) = 5^{4-3x}$

   (b) $h(x) = \log_3(1 - x^2)$

2. (15) If $y = e^{xy}$, find $\dfrac{dy}{dx}$.

3. (22) For each of the following, decide if it possible to use L'Hôpital's Rule. If you can, evaluate the limit by using it.

   (a) YES NO $\lim\limits_{x\to 0} \dfrac{x}{\sin(x)}$

   (b) YES NO $\lim\limits_{x\to \infty} \dfrac{x}{\sin(x)}$

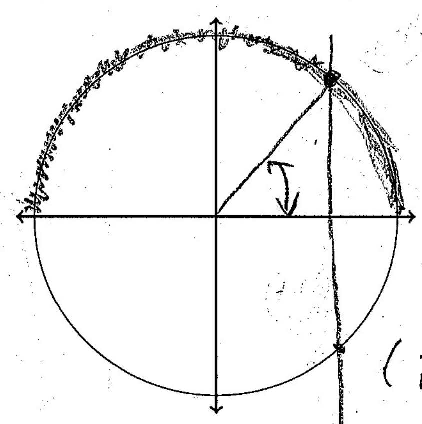
   (c) YES NO $\lim\limits_{x\to -\infty} \dfrac{e^x}{x}$

   (d) YES NO $\lim\limits_{x\to \infty} \dfrac{e^x}{x}$

4. (20) Evaluate each of the following. You can *check* with your calculator, but when appropriate, you **must** include each of the following on the unit circle: (1) the relevant point and the coordinates of that point, (2) the appropriate line on the unit circle, (3) the equation of the line next to it. Note: If you do *not* need the unit circle, you do not have to draw on it. You have to decide.

   (a) $\arccos(\cos(7\pi/4))$

$= \frac{\pi}{4}$

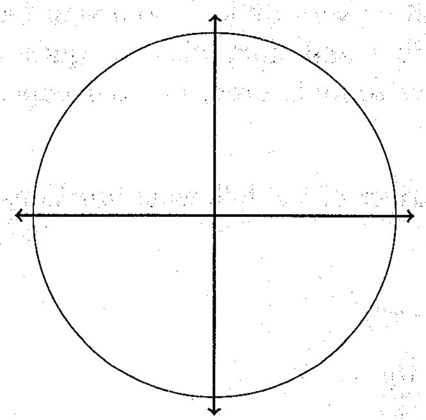


$\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$x = \frac{1}{\sqrt{2}}$

(b) $\arctan(\tan(-\pi/3))$

$= -\frac{\pi}{3}$



5. (7) Find $\frac{d}{dx}\arcsin(1-x)$. Simplify as much as possible.

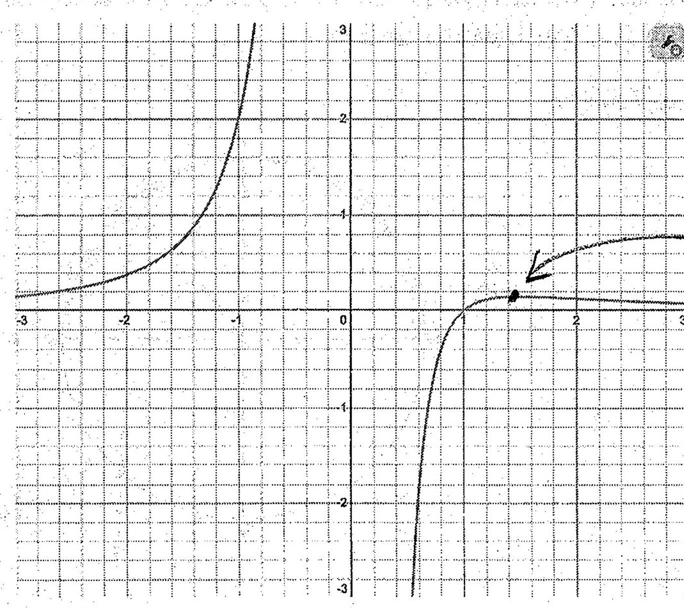
6. (7) Find $\lim_{x\to 0^-} \frac{2-3x}{x\sin(x)}$.

7. (17) Consider $f(x) = \frac{x-1}{x^3}$. Here are the derivatives:

$$f'(x) = \frac{3-2x}{x^4}, \quad f''(x) = \frac{6(x-2)}{x^5}.$$

(a) Find the x-values and y-values of the local maxima and minima. Label them on the graph. You do **not** have to show they are minima. Just label them on the graph.

(b) By creating a **sign chart,** find the intervals where $f(x)$ is concave up and concave down.



$\left(\frac{3}{2}, \frac{4}{27}\right)$