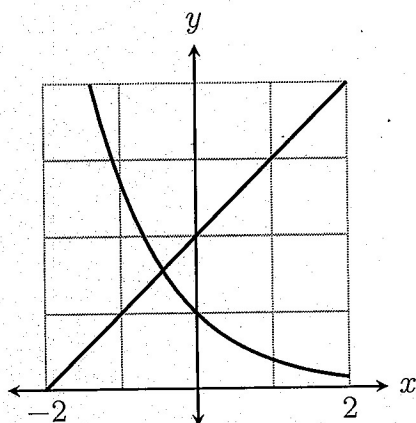


- +10 1. Show that the graphs of $f(x) = e^{-x}$ and $f(x) = x + 2$ intersect somewhere in the interval $[-2, 2]$.



$$h(x) = e^{-x} - (x + 2) \\ = e^{-x} - x - 2$$

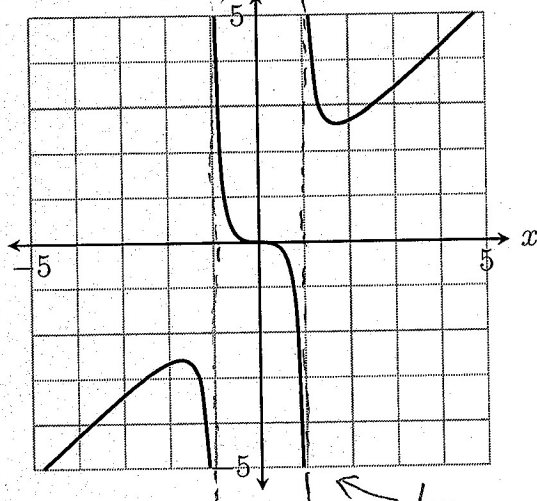
$$h(-2) \approx 7.39$$

$$h(2) \approx -3.86$$

Since $-3.86 < 0 < 7.39$, there is x_0 in $(-2, 2)$ with $h(x_0) = 0$.

- +15 2. Below is a graph of $y = \frac{x^3}{x^2 - 1}$. Find all asymptotes, sketch them on the graph, and label the behavior near the asymptotes using the appropriate limit notation.

$$\lim_{x \rightarrow -1^+} f(x) \text{ DNE } (+\infty) \quad \lim_{x \rightarrow 1^+} f(x) \text{ DNE } (+\infty)$$



$$N = 3, D = 2$$

$$N > D : N_0 \text{ H.A.}$$

$$x^2 - 1 = 0$$

$$x = \pm 1 \text{ V.A.}$$

$$\lim_{x \rightarrow -1^-} f(x) \text{ DNE } (-\infty) \quad \lim_{x \rightarrow 1^-} f(x) \text{ DNE } (-\infty)$$

+10 3. Find $\lim_{x \rightarrow -\infty} \frac{1-x^2}{e^x-1}$. Of the form $\frac{-\infty}{-1}$

DNE ($+\infty$)

+10 4. Find $\lim_{x \rightarrow \infty} \frac{1-x^2}{e^x-1}$. Of the form $\frac{-\infty}{\infty}$. Use LR.

$$\lim_{x \rightarrow \infty} \frac{1-x^2}{e^x-1} \stackrel{\text{LR}}{=} \lim_{x \rightarrow \infty} \frac{-2x}{e^x} \stackrel{\text{LR}}{=} \lim_{x \rightarrow \infty} \frac{-2}{e^x} = 0$$

+10 5. Find $\frac{d}{dx} 3^{2x+1}$. $f(x) = 3^x$ $f'(x) = 3^x \ln 3$
 $g(x) = 2x+1$ $g'(x) = 2$

$$f'(g(x)) g'(x) \\ 3^{g(x)} \ln 3 \cdot 2 = 3^{2x+1} (2 \ln 3)$$

+10 6. Find $\frac{d}{dx} x \log_2(x)$. $f(x) = x$ $g(x) = \log_2(x)$
 $f'(x) = 1$ $g'(x) = \frac{1}{x \ln 2}$

$$f(x) g'(x) + g(x) f'(x)$$

$$x \cdot \frac{1}{x \ln 2} + \log_2(x) \cdot 1$$

$$\frac{1}{\ln 2} + \log_2(x)$$

+10 7. Find $\lim_{x \rightarrow \infty} (x+3)e^{-x}$. Rewrite as $\lim_{x \rightarrow \infty} \frac{x+3}{e^x} = \frac{\infty}{\infty}$ Use LR

$$\lim_{x \rightarrow \infty} \frac{x+3}{e^x} \stackrel{LR}{=} \lim_{x \rightarrow \infty} \frac{1}{e^x} = 0$$

+10 8. Given the curve described by the equation $x^3 - 3xy = y^2 + 1$, find $\frac{dy}{dx}$.

$$\frac{d}{dx} x^3 - \frac{d}{dx} 3xy = \frac{d}{dx} y^2 + 1$$

$$3x^2 - 3\left(x \frac{dy}{dx} + y\right) = 2y \frac{dy}{dx}$$

$$3x^2 - 3x \frac{dy}{dx} - 3y = 2y \frac{dy}{dx}$$

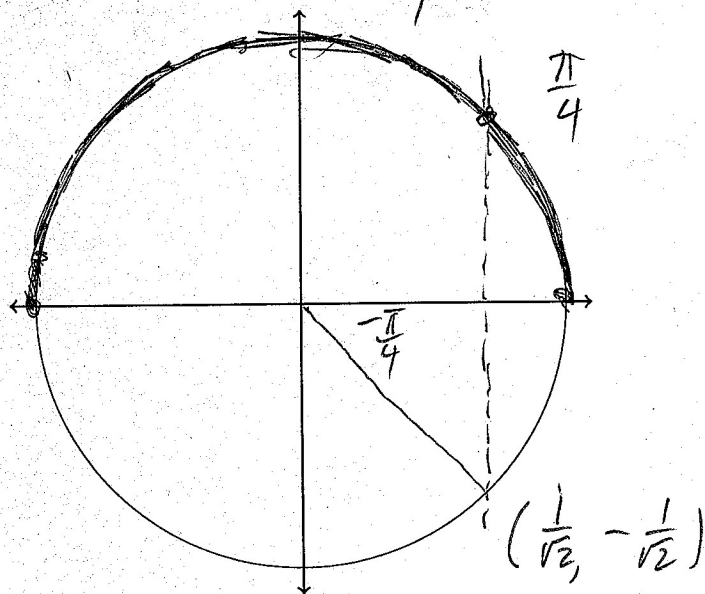
$$+ 3x \frac{dy}{dx} \qquad + 3x \frac{dy}{dx}$$

$$3x^2 - 3y = 2y \frac{dy}{dx} + 3x \frac{dy}{dx}$$

$$3x^2 - 3y = \frac{dy}{dx} (2y + 3x)$$

$$\frac{dy}{dx} = \frac{3x^2 - 3y}{2y + 3x}$$

9. Find $\arccos(\cos(-\pi/4)) = \frac{\pi}{4}$



10. Find $\tan(\arctan(-2)) = -2$

