

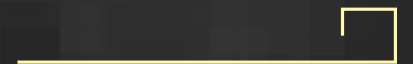
Knights and Rogues

JRMF Problem Incubator

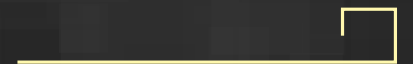
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Vince Matsko
St. Petersburg, FL

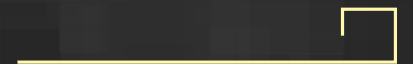
They were dark times in the Kingdom of Verdoon. A band of fierce Rogues was terrorizing the local hamlets. King Buford was very displeased – he had no other option but to send out a legion of Brave Knights and Loyal Knights to round up the Rogues and deliver them to the castle dungeons.



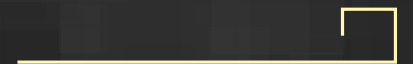
So he chose seven of his bravest Brave Knights, and seven of his most loyal Loyal Knights, and dispatched them across Mount Kerchoo to the hamlet of Dunken, where the Rogues were last reported to have been wreaking havoc.



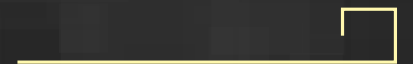
The Knights were capable Knights, and soon rounded up seven of the fiercest Rogues, including their leader, Thorn Yackley. Now they had the difficult task of escorting the Rogues back through the narrow, winding path over Mount Kerchoo.



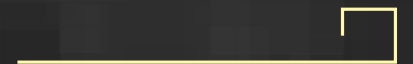
The Rogues were so fierce, no single sword could subdue any of them, nor could a single mace. But they were no match for both a sword and a mace.



Now the Brave Knights carried a single sword, and the Loyal Knights a single mace. So on the trek back to the castle – where they had to walk single file, the path was so narrow – it was vital that each Rogue be flanked by both a Brave Knight and a Loyal Knight in the event the Rogue tried to escape.



The question: In how many ways could the Brave Knights, Loyal Knights, and the Rogues be lined up so that on the treacherous way back, each Rogue was flanked by both a Brave Knight and a Loyal Knight?



Problem statement, Knights and Rogues:

Given p 0's, q 1's, and r 2's, let $\kappa(p, q, r)$ be the number of strings which may be formed so that each 2 is flanked by both a 0 and a 1.

Constraints:

$$p + q > r$$

$$p, q \geq \left\lceil \frac{r}{2} \right\rceil$$



$\kappa(2, 2, 2)$

021 + 021 2×2

02120 + 1 2

12021 + 0 2

Total 8



$\kappa(2, n, 2)$

$$021 + 021 + (n - 2) \text{ 1's} \quad \binom{n}{2} \times 2 \times 2$$

$$02120 + (n - 1) \text{ 1's} \quad n$$

$$12021 + 0 + (n - 2) \text{ 1's} \quad \binom{n}{2} \times 2$$

$$\text{Total} \quad 3n^2 - 2n$$

Note: $n = 1$ only consists of the middle case.



Suggested problems:

1. Find $\kappa(1, 5, 1)$.
2. Find $\kappa(2, 3, 2)$.
3. Find $\kappa(3, 3, 3)$.
4. Find $\kappa(1, n, 2)$, $n \geq 2$.
5. Find $\kappa(1, n, 1)$, $n \geq 1$.
6. Find $\kappa(2, n, 1)$, $n \geq 1$.
7. Find $\kappa(n, n, 1)$, $n \geq 1$.
8. Find $\kappa(3, n, 3)$, $n \geq 2$.
9. Find $\kappa(n, n, 2)$, $n \geq 1$.
10. Why are the $\kappa(2, n, 2)$ the octagonal numbers?
11. Extend to two-parameter families.
12. Adapt this problem to a JRMF activity.

