

1. Find the slope of the tangent line to the hyperbola $y = \frac{6}{x}$ at $x = 2$.

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} &= \lim_{x \rightarrow 2} \frac{\frac{6}{x} - 3}{x - 2} = \lim_{x \rightarrow 2} \frac{\frac{6}{x} - \frac{3 \cdot x}{1 \cdot x}}{x - 2} \\ &= \lim_{x \rightarrow 2} \frac{\frac{6 - 3x}{x}}{x - 2} = \lim_{x \rightarrow 2} \frac{6 - 3x}{x} \cdot \frac{1}{x - 2} \\ &= \lim_{x \rightarrow 2} \frac{-3(x - 2)}{x} \cdot \frac{1}{x - 2} = \lim_{x \rightarrow 2} \frac{-3}{x} = -\frac{3}{2} \end{aligned}$$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} &= \lim_{h \rightarrow 0} \frac{\frac{6}{2+h} - 3}{h} = \lim_{h \rightarrow 0} \frac{\frac{6}{2+h} - 3 \cdot \frac{2+h}{2+h}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{6 - 3(2+h)}{2+h}}{h} = \lim_{h \rightarrow 0} \frac{6 - 3(2+h)}{2+h} \cdot \frac{1}{h} \\ &= \lim_{h \rightarrow 0} \frac{6 - 6 - 3h}{2+h} \cdot \frac{1}{h} = \lim_{h \rightarrow 0} \frac{-3h}{2+h} \cdot \frac{1}{h} = \lim_{h \rightarrow 0} \frac{-3}{2+h} = -\frac{3}{2} \end{aligned}$$

2. Suppose you are given that $f(x) = 2x^3 - 6x^2$ and $f'(1) = -6$. Find the equation of the tangent line through $(1, -4)$.

Need point and slope

↑
(1, -4)

↑
-6

$$y = mx + b$$

$$y = -6x + b$$

$$-4 = -6(1) + b$$

$$-4 = -6 + b$$

$$2 = b$$

$$y = -6x + 2$$

3. The height in feet of a ball thrown into the air at time t is given by $y = 20t - 16t^2$. Find the average velocity for the time period beginning when $t = 2$ and lasting 1 second.

$$\text{average velocity} = \frac{\text{change in position}}{\text{time elapsed}}$$

$$= \frac{y(3) - y(2)}{3 - 2}$$

$$= \frac{-84 - (-24)}{1}$$

$$= -60 \text{ ft/s}$$

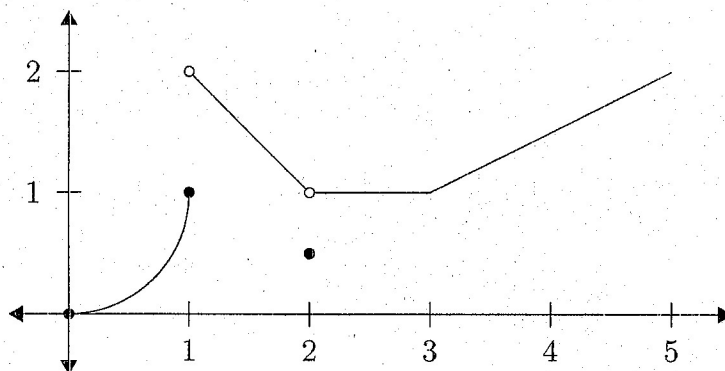
$$y(3) = 20 \cdot 3 - 16 \cdot 3^2$$

$$= 60 - 144 = -84$$

$$y(2) = 20 \cdot 2 - 16 \cdot 2^2$$

$$= 40 - 64 = -24$$

4. This is the graph of $y = f(x)$ on the domain $0 \leq x \leq 5$.



Find the following limits. If a limit does not exist, write "DNE." You do not have to show work.

$$(a) \lim_{x \rightarrow 1^+} f(x) = 2$$

$$(b) \lim_{x \rightarrow 1^-} f(x) = 1$$

$$(c) \lim_{x \rightarrow 1} f(x) = \text{DNE (does not exist)}$$

$$(d) f(2) = \frac{1}{2}$$

$$(e) \lim_{x \rightarrow 2} f(x) = 1$$