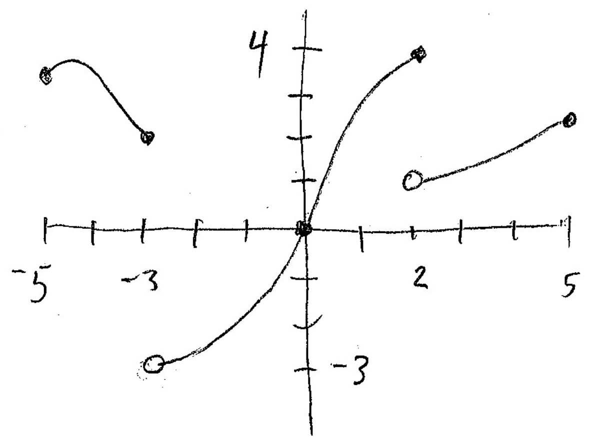
Practice Exam, Solutions

1) $\lim_{x \to 4} \frac{2-\sqrt{x}}{x-4} = \lim_{x \to 4} \frac{(2-\sqrt{x})(2+\sqrt{x})}{(x-4)(2+\sqrt{x})}$

$= \lim_{x \to 4} \frac{4+2\sqrt{x}-2\sqrt{x}-x}{(x-4)(2+\sqrt{x})} = \lim_{x \to 4} \frac{4-x}{(x-4)(2+\sqrt{x})}$

$= \lim_{x \to 4} \frac{-1\cancel{(x-4)}}{\cancel{(x-4)}(2+\sqrt{x})} = \lim_{x \to 4} \frac{-1}{2+\sqrt{x}} = \frac{-1}{2+\sqrt{4}} = -\frac{1}{4}$

2) Other correct answers are possible.



3) f is continuous when $x \neq 0$ since polynomials are continuous everywhere.

$x=0$: $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 = 0$

$x=1$: $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x-1) = -1.$

Since $0 \neq -1$, f is <u>not</u> continuous at 0.

Since $f(0) = -1$, f is continuous from the right at 0.

4) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{6}{x+h} - \frac{6}{x}}{h}$

$= \lim_{h\to 0} \frac{\frac{6}{x+h}\cdot\frac{x}{x} - \frac{6}{x}\cdot\frac{x+h}{x+h}}{h} = \lim_{h\to 0} \frac{\frac{6x-6(x+h)}{(x+h)x}}{h}$

$= \lim_{h\to 0} \frac{\frac{6x-6x-6h}{(x+h)x}}{h} = \lim_{h\to 0} \frac{-6\cancel{h}}{(x+h)x}\cdot\frac{1}{\cancel{h}}$

$= \lim_{h\to 0} \frac{-6}{(x+h)x} = \frac{-6}{x^2}$ Domain: $(-\infty,0)\cup(0,\infty)$

5) a) $f'(x) = 4\cdot 5x^{5-1} - 3\cdot 3x^{3-1} + \frac{1}{2}x^{\frac{1}{2}-1}$
$= 20x^4 - 9x^2 + \frac{1}{2}x^{-1/2}$

b) $g'(x) = 3\cos(x) - 2(-\sin(x)) = 3\cos(x) + 2\sin(x)$

c) $p'(x) = x\cdot\frac{d}{dx}(\sin(x)) + \sin(x)\frac{d}{dx}(x)$
$= x\cdot\cos(x) + \sin(x)\cdot 1 = x\cos(x) + \sin(x)$

d) $q'(x) = \frac{(x^2+1)\frac{d}{dx}(x+2) - (x+2)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$

$= \frac{(x^2+1)\cdot 1 - (x+2)\cdot 2x}{(x^2+1)^2}$

$= \frac{x^2+1-2x^2-4x}{(x^2+1)^2} = \frac{-x^2-4x+1}{(x^2+1)^2}$

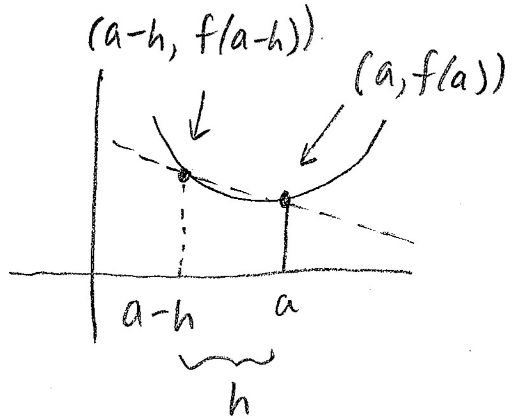
6) $\lim_{x \to -\infty} \dfrac{1 - x - x^2}{4 + x + x^2} = \lim_{x \to -\infty} \dfrac{-x^2}{x^2}$

Only use highest powers in numerator and denominator

$= \lim_{x \to -\infty} -1 = -1.$

The graph has a horizontal asymptote at $y = -1$.

7)



$$\text{Slope} = \frac{f(a) - f(a-h)}{a - (a-h)} = \frac{f(a) - f(a-h)}{h}.$$

Cal's idea won't quite work, it gives the negative of the slope. The numerator should be $f(a) - f(a-h)$.

8) As $x \to -\infty$, then $\frac{1}{x} \to 0$. Thus, $\lim_{x \to -\infty} \sin\left(\frac{1}{x}\right) = 0$.
As $x \to +\infty$, then $\frac{1}{x} \to 0$. Thus, $\lim_{x \to +\infty} \sin\left(\frac{1}{x}\right) = 0$.
As $x \to 0$, $\frac{1}{x}$ gets larger (both + and −). But looking at the graph of $\sin(x)$, there is no limit (DNE) as $x \to -\infty$ or $x \to +\infty$ since the graph oscillates.

9) Let $f(x) = \frac{1}{x}$, $g(x) = 1 - \frac{1}{x}$. Then $\lim_{x \to 0^+} \frac{1}{x}$ DNE $(+\infty)$ and $\lim_{x \to 0^-} \frac{1}{x}$ DNE $(-\infty)$. So $\lim_{x \to 0} \frac{1}{x}$ does not exist.
Similarly, $\lim_{x \to 0} g(x)$ does not exist. But $f(x) + g(x) = \frac{1}{x} + 1 - \frac{1}{x} = 1$, and so $\lim_{x \to 0} (f(x) + g(x)) = \underline{1}$.