

This homework assignment is based on function composition. Please review Examples 4 and 5 on pp. 19–20 of your book. Here, we expand on Example 6.

Example 1: Suppose that $p(x) = \tan^2(x)$. Find functions f and g such that $p(x) = (f \circ g)(x)$.

Solution: Recall that $(f \circ g)(x) = f(g(x))$. So you apply g *first*, and you apply f *last*. Think about $p(x)$. What do you do first? You take $\tan(x)$. What do you do last? You square it. What you do first is $g(x)$, so $g(x) = \tan(x)$. What you do last is $f(x)$, so $f(x) = x^2$.

Check:

$$(f \circ g)(x) = f(g(x)) = f(\tan(x)) = (\tan(x))^2 = \tan^2(x).$$

Example 2: Suppose you are given that $q(x) = \sqrt{x^2 + 1}$. Find functions f and g such that $q(x) = (f \circ g)(x)$.

Solution: As in the previous example, recall that $(f \circ g)(x) = f(g(x))$. So you apply g *first*, and *then* you apply f . Think about $q(x)$. What do you do first? You use x to find $x^2 + 1$. Then what do you do? You take the square root. What you do first is $g(x)$, so $g(x) = x^2 + 1$. Then you apply $f(x)$, so $f(x) = \sqrt{x}$.

Check:

$$(f \circ g)(x) = f(g(x)) = f(x^2 + 1) = \sqrt{x^2 + 1}.$$

Example 3: Suppose you are given that $q(x) = \frac{1}{3 + \cos(x)}$. Find functions f and g such that $q(x) = (f \circ g)(x)$.

Solution: In this example, you can see that there can be more than one way to do this. Both ways are correct.

Method 1: Put $g(x) = 3 + \cos(x)$, $f(x) = \frac{1}{x}$. Then

$$(f \circ g)(x) = f(g(x)) = f(3 + \cos(x)) = \frac{1}{3 + \cos x}.$$

Method 2: Put $g(x) = \cos(x)$, $f(x) = \frac{1}{3 + x}$. Then

$$(f \circ g)(x) = f(g(x)) = f(\cos(x)) = \frac{1}{3 + \cos(x)}.$$

Now it's your turn! For each of the following functions, find appropriate functions f and g so that the given function is $(f \circ g)(x)$. Keep in mind that your answer may be different than mine and still be correct. Answers are on the next page.

1. $p(x) = \tan(3x + 1)$

2. $q(x) = (1 + \sin(x))^5$

3. $r(x) = \cos(\sqrt{x})$

4. $s(x) = \sqrt{3 - \sqrt{x}}$

5. $t(x) = \frac{1 + \tan(x)}{5 \tan(x)}$

6. $u(x) = \left(\frac{x^2 + 1}{x}\right)^3$

Answers:

1. $f(x) = \tan(x)$, $g(x) = 3x + 1$.

2. $f(x) = x^5$, $g(x) = 1 + \sin(x)$.

3. $f(x) = \cos(x)$, $g(x) = \sqrt{x}$.

4. $f(x) = \sqrt{x}$, $g(x) = 3 - \sqrt{x}$.

5. $f(x) = \frac{1+x}{5x}$, $g(x) = \tan(x)$.

6. $f(x) = x^3$, $g(x) = \frac{x^2 + 1}{x}$.