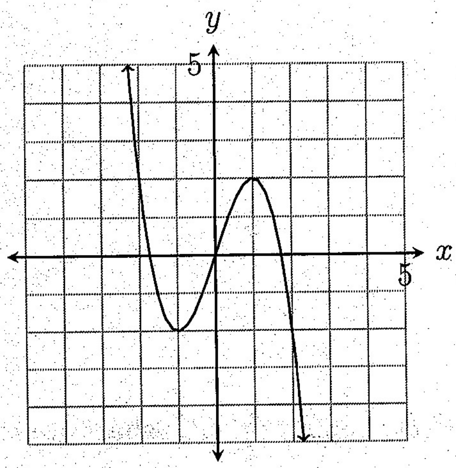
Homework Quiz 3, 6 Mar 2024 MA 131M

1. Below is a graph of $f(x) = 3x - x^3$. Then $f'(x) = 3 - 3x^2$. By making a sign chart for $f'(x)$, find all local minima and maxima. All work must be shown; use the graph for verification only.



+8

$f'(x) = 3 - 3x^2 = 0$

$3x^2 = 3$

$x^2 = 1, \quad x = \pm 1$

$-$ $+$ $-$

-1 $+1$

$f(-1) = 3(-1) - (-1)^3 = -2$

$f(1) = 3 \cdot 1 - 1^3 = 2$

Local min : $(-1, -2)$ Local max $(1, 2)$

2. Using the same function as above, find all inflection points by making a sign chart for $f''(x)$.

+4

$f''(x) = -6x = 0$

$x = 0$

$f''(x)$ $+$ $-$

0

$f(0) = 0$,   Inflection pt. at $(0, 0)$.

3. If $h(x) = e^x \cos(x)$, find $h'(x)$.   Product Rule

$f(x) = e^x \qquad f'(x) = e^x$

$g(x) = \cos(x) \qquad g'(x) = -\sin(x)$

+6 $f(x)\,g'(x) + g(x)\,f'(x)$

$e^x \cdot (-\sin(x)) + \cos(x) \cdot e^x$

$-e^x \sin(x) + e^x \cos(x)$

4. If $h(x) = x^3 \ln(x)$, find $h'(x)$. Product Rule

$f(x) = x^3$ $f'(x) = 3x^2$

+6 $g(x) = \ln(x)$ $g'(x) = \frac{1}{x}$

$f(x)g'(x) + g(x)f'(x)$

$x^3 \cdot \frac{1}{x} + \ln(x) \cdot 3x^2$

$x^2 + 3x^2 \ln(x)$

5. Suppose a population of bacteria is modeled by $P(t) = 8000e^{0.01t}$. At what rate is the population increasing at 4 hours?

$P'(t) = 8000\, e^{0.01t} (.01) = 80 e^{0.01t}$

$P'(4) = 83.26$

+5

Population increasing at 84 bacteria per hour.

6. We see from the graph of $y = \ln(x)$ that this function is increasing on its domain. Show this using calculus.

$\frac{dy}{dx} \ln(x) = \frac{1}{x}$ Domain of $\ln(x)$ is $(0, \infty)$.

+5 So $\frac{1}{x} > 0$ for all x in the domain.

So the graph is always increasing.