1. Decide if the following are true or false. Circle one.

   (a) (TRUE) FALSE $f : \mathbb{N} \to \mathbb{N}$, $f(x) = x^2$ is injective.
   (b) TRUE (FALSE) $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x^2$ is injective.
   (c) TRUE (FALSE) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ is injective.
   (d) TRUE (FALSE) $f : \mathbb{N} \to \mathbb{N}$, $f(x) = x + 1$ is surjective.
   (e) (TRUE) FALSE $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x + 1$ is surjective.
   (f) (TRUE) FALSE $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x + 1$ is surjective.

2. If $f : \mathbb{N} \to \mathbb{N}$, $f(x) = x^2$, find $f^{-1}(9)$.

   $\{3\}$

3. If $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x^2$, find $f^{-1}(9)$.

   $\{-3, 3\}$

4. Suppose you have sets X and Y such that $|X| = 12$ and $|Y| = 42$.

   (a) What is the largest possible value for $|X \cap Y|$? 12

   (b) What is the smallest possible value for $|X \cap Y|$? 0

   (c) What are the possible values for $|X \cup Y|$? $42 \leq |X \cup Y| \leq 54$

5. Consider all six-letter words made from the letters a, b, and c.

   (a) How many words contain no repeated letters? 0

   (b) How many words begin with a and end with b? $3^4 = 81$

   a _ _ _ _ b

   (c) How many words contain the string "*cccc*" somewhere within the word?

   4 c's: cccc__, _cccc_, __cccc 12
   5 c's: ccccc_, _ccccc, c_cccc, 8
   6 c's: cccccc, ccccc_c 1
   21

6. In how many ways is it possible to draw one card if that card is either a spade or a 4?

$13 + 4 - 1 = 16$

spades; a four; $4\spadesuit$ is overcounted

7. Let $A = \{2, 3, 5, 7, 11, 13\}$.

   (a) How many subsets have cardinality 2? $\binom{6}{2} = 15$

   (b) How many subset contain at least one odd number? 62

All $2^6 = 64$ subsets EXCEPT $\emptyset$, $\{2\}$

   (c) How many subsets have the property that when you add the numbers in the subset, the sum is odd?

You cannot have an even number of odd numbers

$\left[1 + \binom{5}{2} + \binom{5}{4}\right] \cdot 2 = 32$, So $64 - 32 = 32$

1: no odd numbers; $\binom{5}{2}$: subsets with 2 odd nos.; $\binom{5}{4}$: subsets with 4 odd nos; $\cdot 2$: may or may not contain 2

8. How many shortest paths start at $(3, 4)$ and

   (a) end at $(7, 6)$? $\binom{6}{2} = 15$

   (b) end at $(7, 6)$ and pass through $(5, 5)$?

$\binom{3}{1} \cdot \binom{3}{1} = 9$