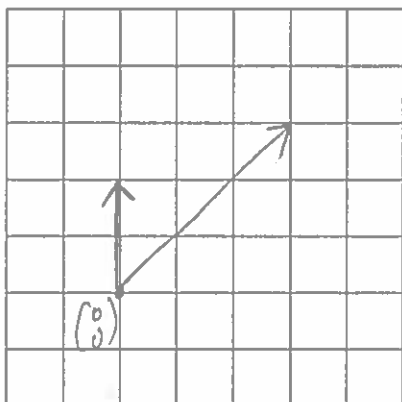


- +5 1. Find a vector  $\mathbf{v}$  such that the line  $-7x + 3y = 0$  lies along  $\mathbf{v}$ .

$$3y = 7x$$

$$y = \frac{7}{3}x \quad \begin{pmatrix} 3 \\ 7 \end{pmatrix} \quad (\text{or a vector parallel to this})$$

- +8 2. Find the angle between  $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$  using the formula learned in class. Sketch!



$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| |\mathbf{v}|} = \frac{\begin{pmatrix} 3 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ 3 \end{pmatrix} \right| \left| \begin{pmatrix} 0 \\ 2 \end{pmatrix} \right|}$$

$$= \frac{6}{\sqrt{18} \cdot 2} = \frac{6}{3\sqrt{2} \cdot 2} = \frac{1}{\sqrt{2}}$$

So,  $\theta = 45^\circ$  or  $\frac{\pi}{4}$ .

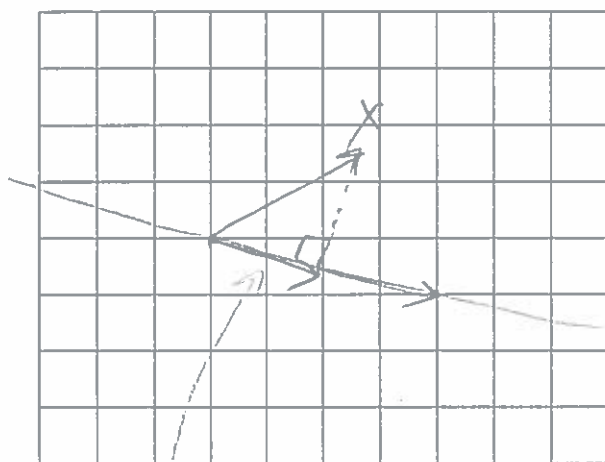
- +5 3. Find all  $x$  such that the vectors  $\begin{pmatrix} x \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 9 \\ x \end{pmatrix}$  are linearly dependent.

$$\frac{x}{9} = \frac{1}{x}$$

$$x^2 = 9$$

$$x = \pm 3$$

- +10 4. Find a formula for the projection onto the line in the direction of  $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ . Sketch an example.



$$P_{\begin{pmatrix} 4 \\ -1 \end{pmatrix}}(x)$$

$$P_{\begin{pmatrix} 4 \\ -1 \end{pmatrix}} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{\begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \end{pmatrix}}{\begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \end{pmatrix}} \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$= \frac{4x - y}{17} \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{16x - 4y}{17} \\ \frac{-4x + y}{17} \end{pmatrix} \quad \text{or}$$

$$\begin{bmatrix} \frac{16}{17} & -\frac{4}{17} \\ -\frac{4}{17} & \frac{1}{17} \end{bmatrix} \text{ in matrix form}$$

- +6 5. You are given that the projection onto the line in the direction of  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$  is  $\begin{pmatrix} (9x - 6y)/13 \\ (-6x + 4y)/13 \end{pmatrix}$ . Find the reflection about this line. Simplify the algebra!

$$S(x) = 2P(x) - X = 2 \begin{pmatrix} \frac{9x - 6y}{13} \\ \frac{-6x + 4y}{13} \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} \frac{18x}{13} - \frac{12y}{13} - x \\ \frac{-12x}{13} + \frac{8y}{13} - y \end{pmatrix} = \begin{pmatrix} \frac{5x}{13} - \frac{12y}{13} \\ \frac{-12x}{13} - \frac{5y}{13} \end{pmatrix}$$