Note: Write ALL affine transformations in terms of the linear part and the translation!

1. Write the affine transformation $f\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x-3+7y\\-6+y\end{pmatrix}$ using its linear part and its translation.

$$f\begin{pmatrix}x\\y\end{pmatrix} = \begin{bmatrix}1 & 7\\0 & 1\end{bmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}-3\\-6\end{pmatrix}$$

2. If $f\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}y\\2-x\end{pmatrix}$ and $g\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-x-1\\-y\end{pmatrix}$, find $f\circ g\begin{pmatrix}x\\y\end{pmatrix}$ and $g\circ f\begin{pmatrix}x\\y\end{pmatrix}$.

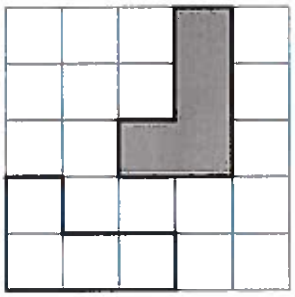
$$f\circ g\begin{pmatrix}x\\y\end{pmatrix} = f\left(g\begin{pmatrix}x\\y\end{pmatrix}\right) = f\begin{pmatrix}-x-1\\-y\end{pmatrix} = \begin{pmatrix}-y\\2-(-x-1)\end{pmatrix} = \begin{pmatrix}-y\\x+3\end{pmatrix}$$

$$g\circ f\begin{pmatrix}x\\y\end{pmatrix} = g\left(f\begin{pmatrix}x\\y\end{pmatrix}\right) = g\begin{pmatrix}y\\2-x\end{pmatrix} = \begin{pmatrix}-y-1\\-(2-x)\end{pmatrix} = \begin{pmatrix}-y-1\\x-2\end{pmatrix}$$

3. Multiply $\begin{bmatrix}3 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}6 & -1\\4 & 2\end{bmatrix}$.

$$\begin{bmatrix}14 & -5\\16 & 0\end{bmatrix}$$

4. Write an affine transformation which transforms the white shape into the gray shape. Assume that the origin is at the lower left corner of the grid.



Rotate white shape 90° CCW, then shift

$$A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

5. Write the affine transformation which moves to the left 4 units, then rotates by 90°, and then moves up 3 units.

left 4: $\begin{pmatrix} x-4 \\ y \end{pmatrix}$

Rotate 90°: $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} x-4 \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x-4 \end{pmatrix}$

up 3: $\begin{pmatrix} -y \\ x-4+3 \end{pmatrix} = \begin{pmatrix} -y \\ x-1 \end{pmatrix}$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

6. If $A = \begin{bmatrix} 1 & 7 \\ -3 & -4 \end{bmatrix}$, find $\det A$.

$1(-4) - (-3)7 = 17$

Note: Write ALL affine transformations in terms of the linear part and the translation!

1. Write the affine transformation $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y - 7 + 5x \\ -2 + x \end{pmatrix}$ using its linear part and its translation.

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 5 & 1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -7 \\ -2 \end{pmatrix}$$

2. If $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ 3 - x \end{pmatrix}$ and $g\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 - x \\ y \end{pmatrix}$, find $f \circ g\begin{pmatrix} x \\ y \end{pmatrix}$ and $g \circ f\begin{pmatrix} x \\ y \end{pmatrix}$.

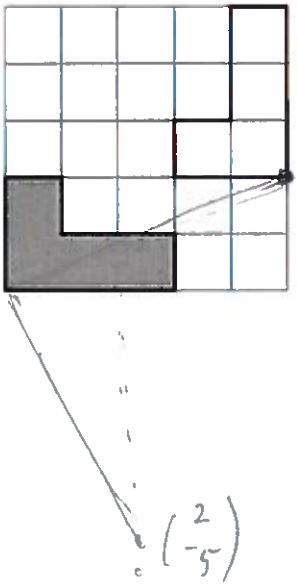
$$f \circ g\begin{pmatrix} x \\ y \end{pmatrix} = f\left(g\begin{pmatrix} x \\ y \end{pmatrix}\right) = f\begin{pmatrix} 2-x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ 3-(2-x) \end{pmatrix} = \begin{pmatrix} -y \\ x+1 \end{pmatrix}$$

$$g \circ f\begin{pmatrix} x \\ y \end{pmatrix} = g\left(f\begin{pmatrix} x \\ y \end{pmatrix}\right) = g\begin{pmatrix} -y \\ 3-x \end{pmatrix} = \begin{pmatrix} 2-(-y) \\ 3-x \end{pmatrix} = \begin{pmatrix} y+2 \\ 3-x \end{pmatrix}$$

3. Multiply $\begin{bmatrix} -3 & 2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$.

$$\begin{bmatrix} -5 & -11 \\ -3 & 13 \end{bmatrix}$$

4. Write an affine transformation which transforms the white shape into the gray shape. Assume that the origin is at the lower left corner of the grid.



Rotate white shape $-90°$ CCW, then shift.

$$A\begin{pmatrix}x\\y\end{pmatrix} = \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}-2\\5\end{pmatrix}$$

$\begin{pmatrix}2\\-5\end{pmatrix}$

5. Write the affine transformation which moves to the right 2 units, then rotates by $-90°$, and then moves down 5 units.

right 2: $\begin{pmatrix}x+2\\y\end{pmatrix}$

Rotate $-90°$: $\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{pmatrix}x+2\\y\end{pmatrix} = \begin{pmatrix}y\\-x-2\end{pmatrix}$

down 5: $\begin{pmatrix}y\\-x-2-5\end{pmatrix} = \begin{pmatrix}y\\-x-7\end{pmatrix}$

$$\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}0\\-7\end{pmatrix}$$

6. If $A = \begin{bmatrix}-5 & -2\\ 6 & 3\end{bmatrix}$, find $\det A$.

$-5(3) - 6(-2) = -3$