Linear Algebra, Homework Quiz 3A 9 March 2018

1. Write a *matrix* equation for solving the following system of equations, and then find all solutions to this system. Show your steps. No points for solving any other way!

$$-5 - 5x = 3y, \qquad 2y + 4x + 2 = 0.$$

+5 setup

+3 inverse

+2 solve

$5x + 3y = -5$

$4x + 2y = -2$

$$\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5 \\ -2 \end{pmatrix}$$

$$\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}^{-1} = -\frac{1}{2}\begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{2}\begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix}\begin{pmatrix} -5 \\ -2 \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} -4 \\ 10 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

2. Find all eigenvalues and eigenvectors of the reflection about the line $y = 3x$.

+8

2pts each

The line stays the same: $\lambda = 1, \ \underline{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

The perpendicular line gets reflected:

$\lambda = -1, \ \underline{v} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

3. $A = \begin{bmatrix} -2 & 2 & 2 \\ 2 & -2 & -1 \\ -6 & 6 & 5 \end{bmatrix}$ has an eigenvalue of 2. Find the corresponding eigenvector(s).

+5 Setup

+3 system

+2 solve

$$\begin{bmatrix} -2 & 2 & 2 \\ 2 & -2 & -1 \\ -6 & 6 & 5 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$-2x + 2y + 2z = 2x$

$2x - 2y - z = 2y$

$-6x + 6y + 5z = 2z$

$-4x + 2y + 2z = 0 \quad \times 1$

$2x - 4y - z = 0 \quad \times 2$

$-6x + 6y + 3z = 0$

$-4x + 2y + 2z = 0$

$4x - 8y - 2z = 0$

$-6y = 0$

$y = 0$

$-4x + 2(0) + 2z = 0$

$-4x + 2z = 0$

$z = 2x$

$$\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

4. Using your die, find all eigenvalues and eigenvectors of $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

+9

3 pts each

$\lambda = 1, \quad \underline{v} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. No other eigenvalues (90° rotation)

5. Find all eigenvalues and eigenvectors of $\begin{bmatrix} 3 & -2 \\ -3 & -2 \end{bmatrix}$. Show all steps!

$$\det \begin{bmatrix} 3-\lambda & -2 \\ -3 & -2-\lambda \end{bmatrix} = (3-\lambda)(-2-\lambda) - 6$$
$$= \lambda^2 - \lambda - 12$$
$$= (\lambda - 4)(\lambda + 3) = 0$$
$$\lambda = 4, -3$$

+5 λ's

+10

+2 setup

+2 system

+1 solve

$\lambda = 4$: $\begin{bmatrix} 3 & -2 \\ -3 & -2 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 4 \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{aligned} 3x - 2y &= 4x \\ -3x - 2y &= 4y \end{aligned} \Rightarrow \begin{aligned} -x - 2y &= 0 \\ -3x - 6y &= 0 \end{aligned} \qquad \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$\lambda = -3$: $\begin{bmatrix} 3 & -2 \\ -3 & -2 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -3 \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{aligned} 3x - 2y &= -3x \\ -3x - 2y &= -3y \end{aligned} \Rightarrow \begin{aligned} 6x - 2y &= 0 \\ -3x + y &= 0 \end{aligned} \qquad \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

1. Write a *matrix* equation for solving the following system of equations, and then find all solutions to this system. Show your steps. No points for solving any other way!

$$-6 - 2x = 3y, \qquad 2y + 3x - 1 = 0.$$

$2x + 3y = -6$
$3x + 2y = 1$

$$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}^{-1} = -\frac{1}{5}\begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{5}\begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix}\begin{pmatrix} -6 \\ 1 \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -15 \\ 20 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

2. Find all eigenvalues and eigenvectors of the reflection about the line $y = -4x$.

The line stays the same: $\lambda = 1$, $\underline{v} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

The perpendicular line gets reflected:
$\lambda = -1$, $\underline{v} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

3. $A = \begin{bmatrix} -2 & 2 & 2 \\ 2 & -2 & -1 \\ -6 & 6 & 5 \end{bmatrix}$ has an eigenvalue of -1. Find the corresponding eigenvector(s).

$$\begin{bmatrix} -2 & 2 & 2 \\ 2 & -2 & -1 \\ -6 & 6 & 5 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -1\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$-2x + 2y + 2z = -x$
$2x - 2y - z = -y$
$-6x + 6y + 5z = -z$

$-3x + 2y + 2z = 0 \quad \times 1$
$-2x - y - z = 0 \quad \times 2$
$-6x + 6y + 6z = 0$

$-3x + 2y + 2z = 0$
$-4x - 2y - 2z = 0$

$-7x = 0$
$x = 0$
$-6(0) + 6y + 6z = 0$
$6y + 6z = 0$
$y = -z$

$$\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

4. Using your die, find all eigenvalues and eigenvectors of $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

$\lambda = 1,\ \underline{v} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$ No other eigenvalues (90° rotation)

5. Find all eigenvalues and eigenvectors of $\begin{bmatrix} 2 & -2 \\ -3 & -3 \end{bmatrix}$. Show all steps!

$$\det \begin{bmatrix} 2-\lambda & -2 \\ -3 & -3-\lambda \end{bmatrix} = (2-\lambda)(-3-\lambda) - 6$$
$$= \lambda^2 + \lambda - 12$$
$$= (\lambda+4)(\lambda-3) = 0$$
$$\lambda = -4,\ 3$$

$\lambda = -4$: $\begin{bmatrix} 2 & -2 \\ -3 & -3 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -4 \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{matrix} 2x - 2y = -4x \\ -3x - 3y = -4y \end{matrix} \Rightarrow \begin{matrix} 6x - 2y = 0 \\ -3x + y = 0 \end{matrix} \qquad \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$\lambda = 3$: $\begin{bmatrix} 2 & -2 \\ -3 & -3 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 3 \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{matrix} 2x - 2y = 3x \\ -3x - 3y = 3y \end{matrix} \Rightarrow \begin{matrix} -x - 2y = 0 \\ -3x - 6y = 0 \end{matrix} \qquad \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$