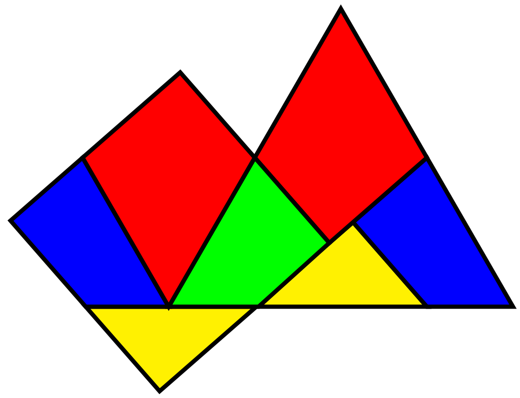
Here is a solution to a classical dissection puzzle: find the fewest number of polygonal pieces which may form *both* an equilateral triangle and a square.



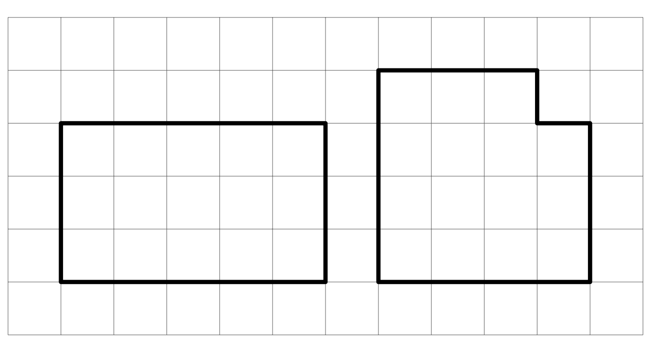
This is a very elegant solution – and students can easily enjoy using the pieces to make one figure, then another.

But where does this solution come from? The mathematics behind the shape of the pieces is somewhat complex, and is certainly far beyond the capabilities of the typical geometry student. As an example, if θ is the angle that the square is rotated relative to the base of the triangle, then $\sin\theta = \sqrt[4]{3}/2$.

This is an example of a geometrical *dissection.* In other words, two geometrical shapes of equal area are cut into exactly the *same* pieces – although in the process of reassembling one shape from another, the pieces may be rotated, or even turned over.

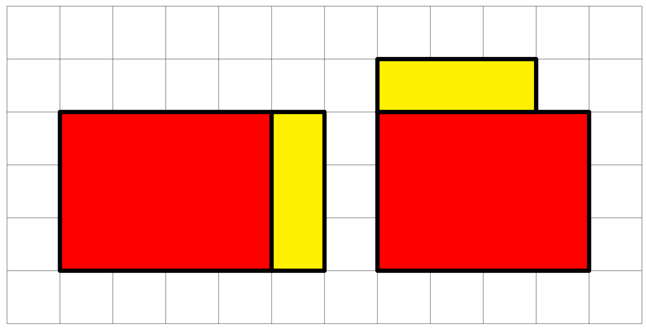
Solving dissection puzzles involves a thinking process very different than that of solving other geometrical problems. It is difficult to make this precise, but if you've worked enough puzzles, you will certainly find this to be true.

It is possible to make such puzzles accessible to younger students by working on a grid. Consider the following two shapes, each of area fifteen (we will use the convention that each square has an area of 1 square unit).



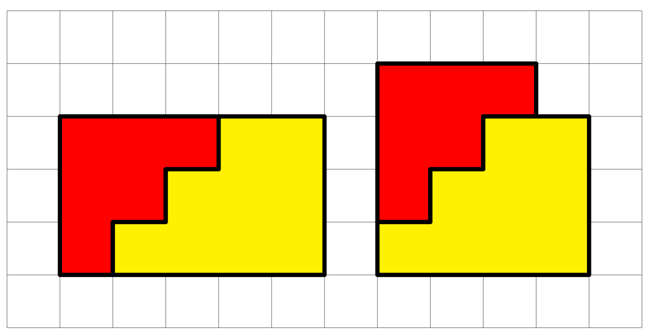
By cutting the rectangle along the grid lines, how many pieces are needed so you can *also* make the square with a corner missing?

This seems like an easy puzzle to solve, as shown below.



So yes, only two pieces are necessary – but one had to be rotated. Here is the question: can this puzzle be solved with just two pieces, but with *neither* piece rotated?

It turns out this is possible – but it requires a bit more creativity. Here is one way to do it:

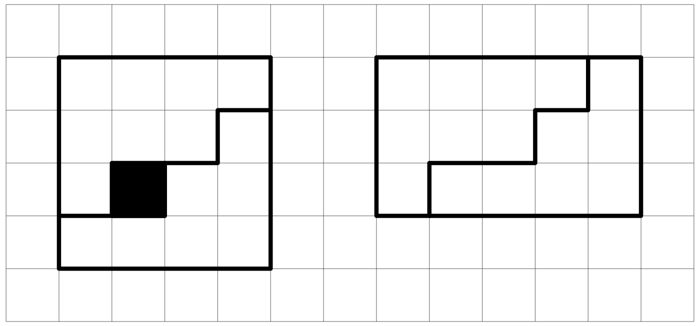


What is not obvious is *how* to come up with such a solution. But by exploring dissections on a grid – which is easy to do with pencil and graph paper – students may be encouraged to think "outside the box." Moreover, imposing additional constraints on a puzzle (such as pieces can not be rotated and/or turned over) can provide challenges for the stronger students, while those who are not so strong may try to find solutions which are not subject to such constraints.

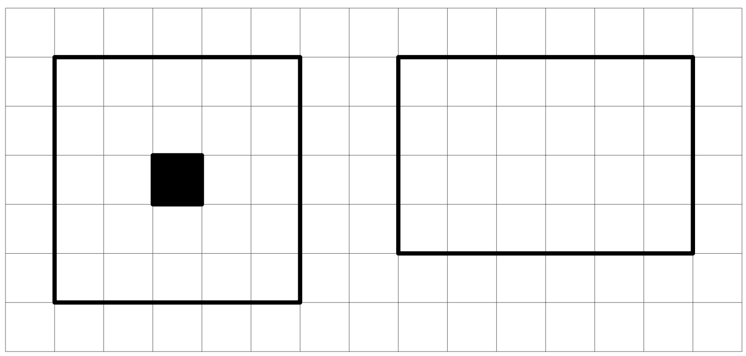
In the next several pages, various dissection puzzles are given. I will give the smallest number of pieces I could find and say whether my solution requires pieces to be rotated or flipped over. This information does not always have to be given, since some students might keep trying to solve a hard puzzle in few pieces and make no progress on other puzzles.

Please let me know if you find a better solution to any of these puzzles! I will include an acknowledgment of your solution in these notes if you let me know.

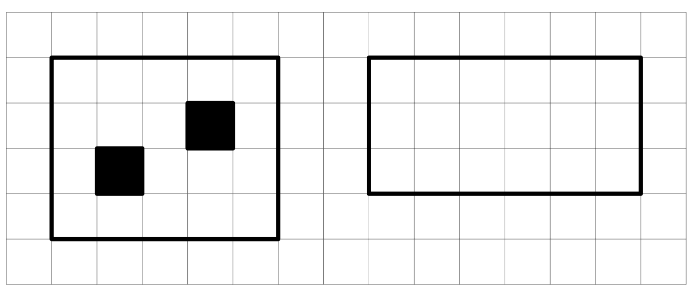
Here is a sample puzzle.



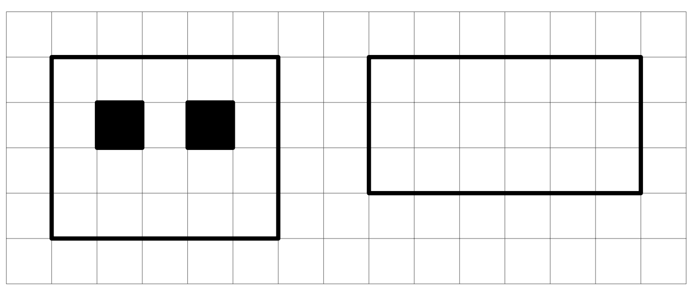
Dissection Puzzle 1: This can be solved with two pieces, neither being rotated or flipped.



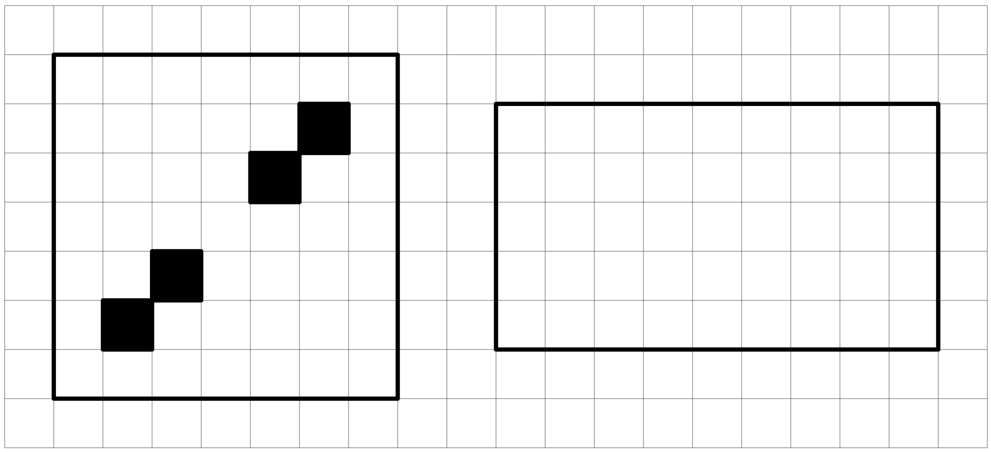
Dissection Puzzle 2: This can be solved with two pieces, neither being rotated or flipped.



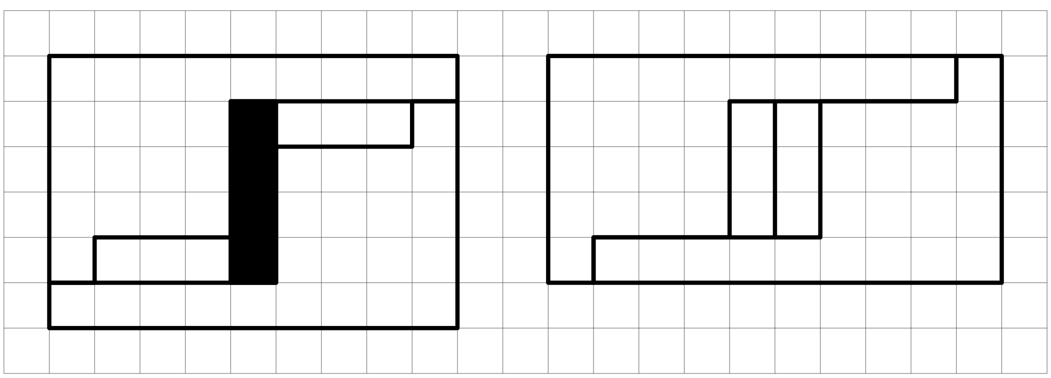
Dissection Puzzle 3: This can be solved with three pieces, none rotated or flipped.



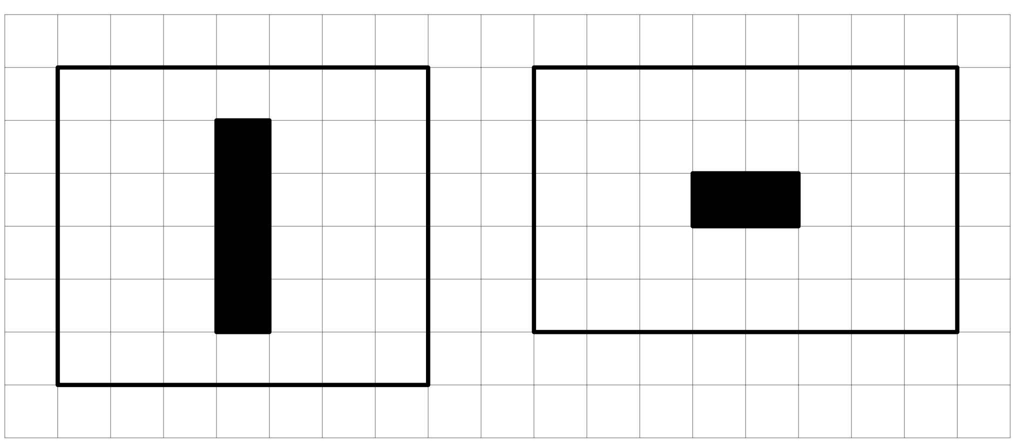
Dissection Puzzle 4: This is a challenging puzzle. It can be solved with four pieces, none rotated or flipped.



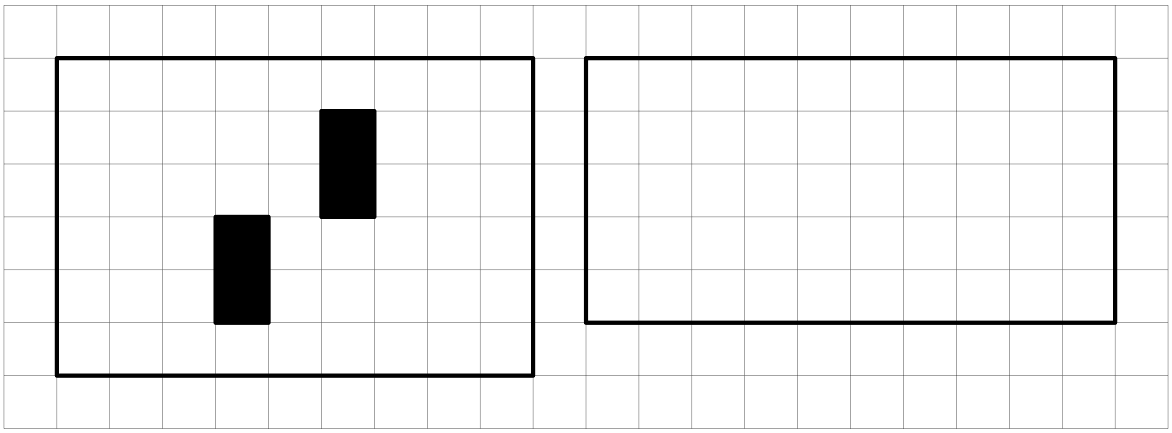
Here is another sample puzzle.



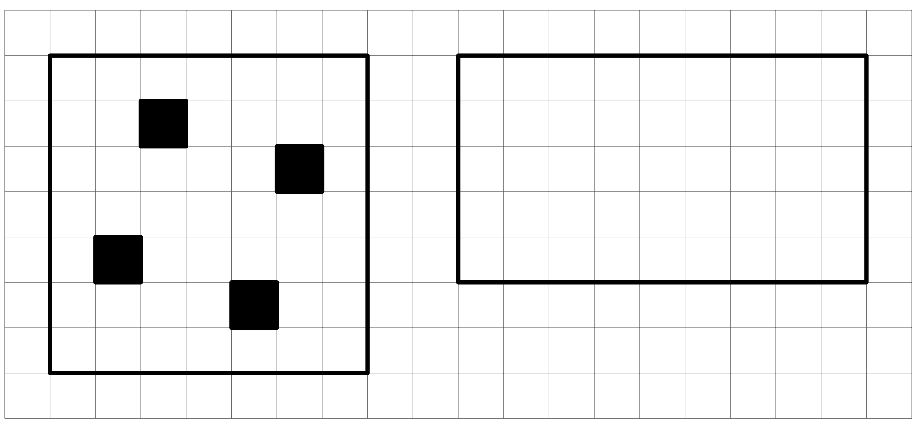
Dissection Puzzle 5: This can be solved with four pieces, none rotated or flipped.



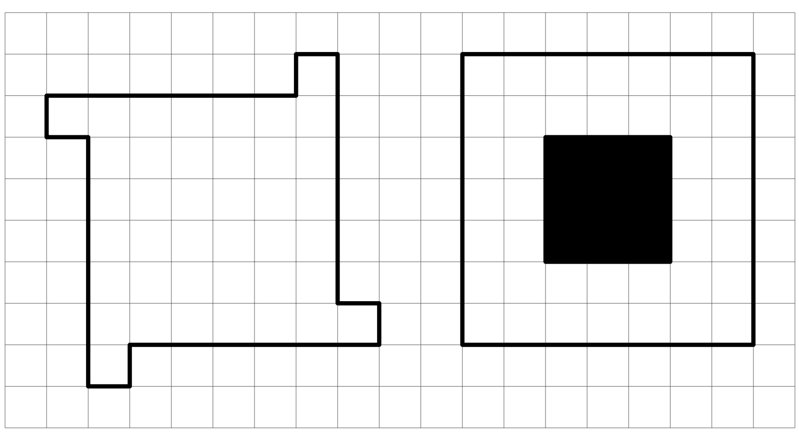
DISSECTION PUZZLE 6: This can be solved with four pieces, none rotated or flipped.



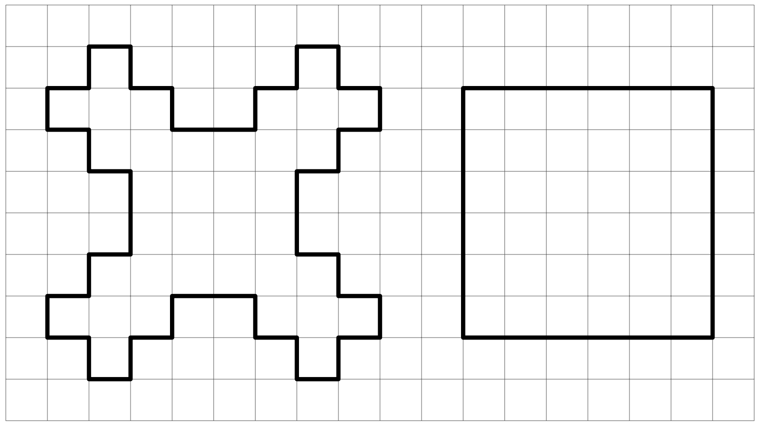
DISSECTION PUZZLE 7: This can be solved in four pieces, with some rotated and/or flipped.



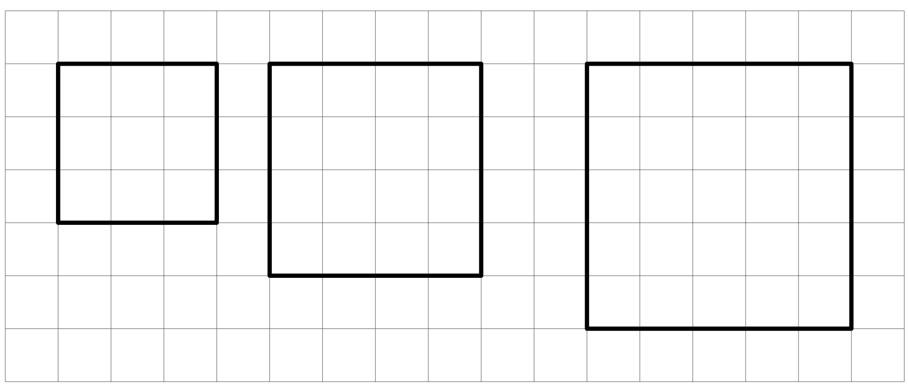
DISSECTION PUZZLE 8: This may be solved with four pieces, some rotated and/or flipped.



Dissection Puzzle 9: This can be solved with six pieces, with one being rotated.

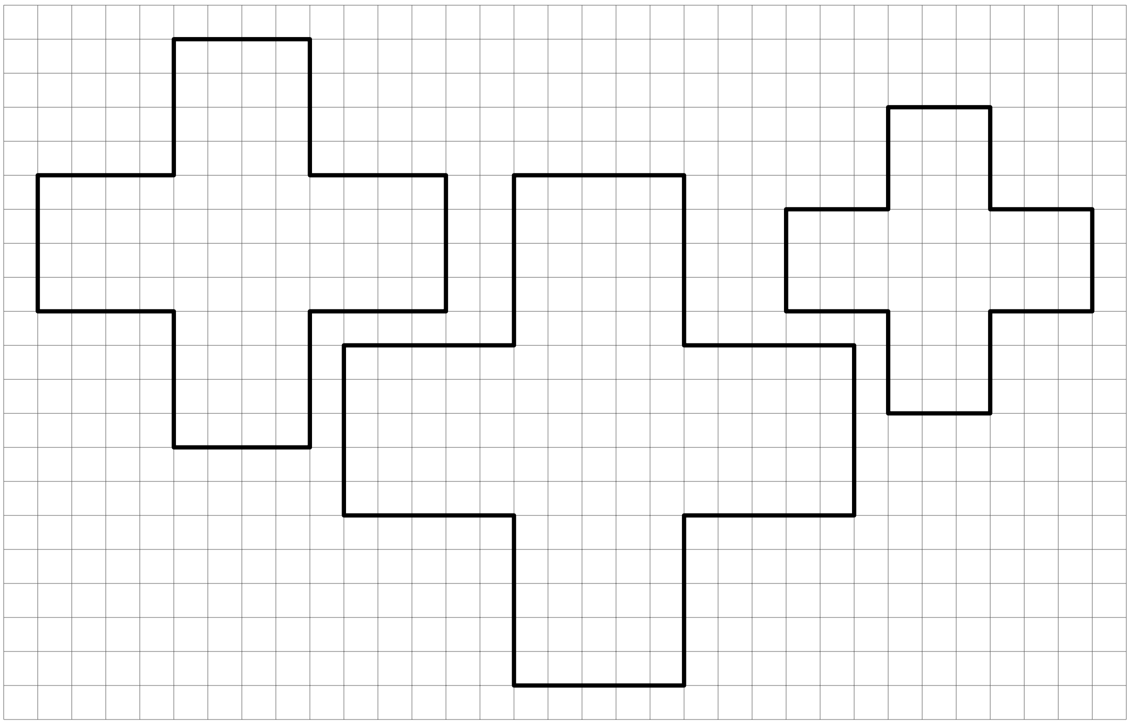


Dissection Puzzle 10: Create the 5×5 square from the two smaller squares. This is one of many examples which can be created using a Pythagorean triple. A four-piece solution with no rotations or flips is attributed to puzzlist Sam Loyd.

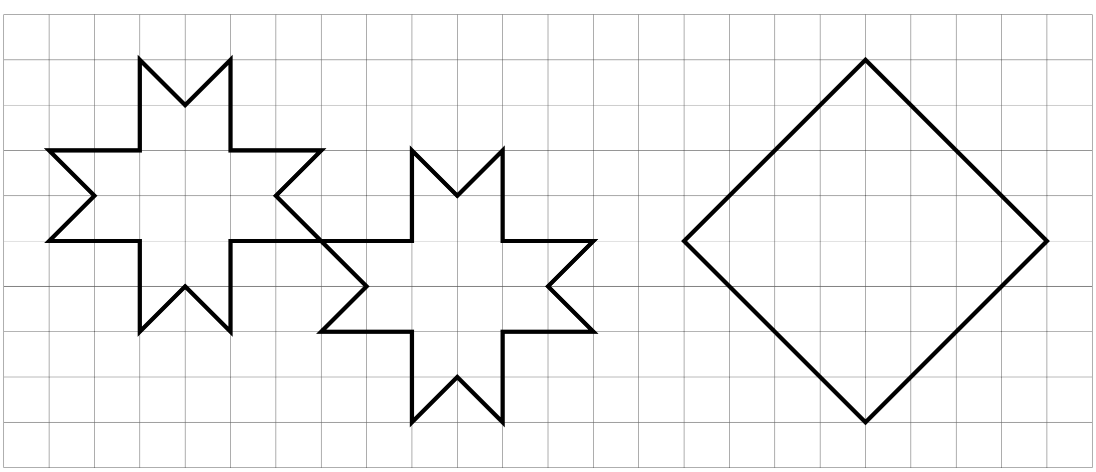


This solution may be found on p. 62 of *Dissections: Plane & Fancy,* by Greg Frederickson, Cambridge University Press, 1997, ISBN 0-521-57197-9. This is a very good book on dissections, but is concerned more with regular figures. Many of the dissections are very beautiful, but are too complex for the typical geometry student.

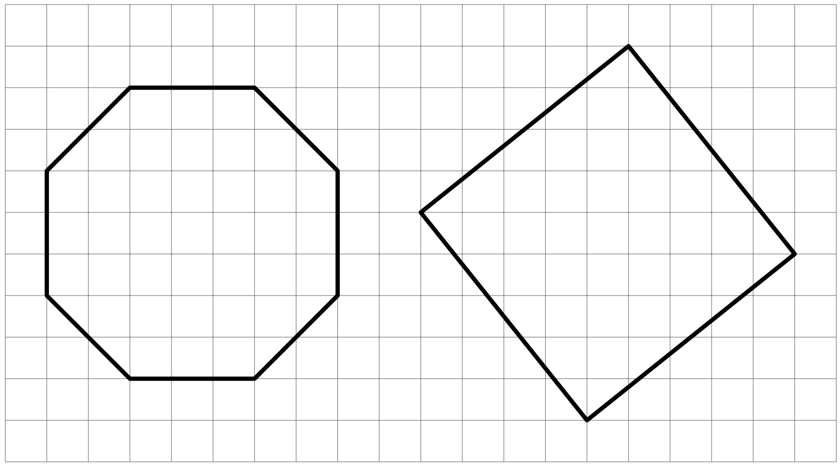
Dissection Puzzle 11: This is a challenging puzzle. Create the large cross from the two smaller ones using eight pieces, some of which are rotated and/or flipped.



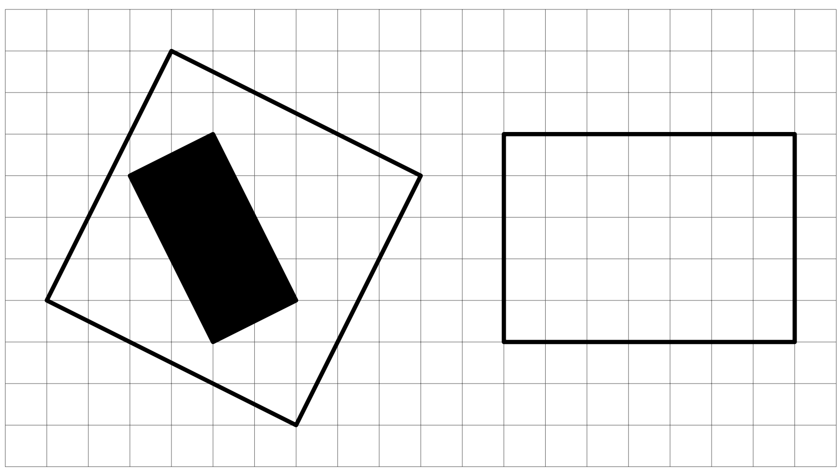
Dissection Puzzle 12: There are many interesting problems which do not only use the grid lines; some examples follow. In this puzzle, create the square from the two stars. This may be solved using eight pieces, with none rotated or flipped.



Dissection Puzzle 13: This may be solved with six pieces, some rotated and/or flipped.



Dissection Puzzle 14: This may be solved with five pieces, some rotated and/or flipped.



Dissection Puzzle 15: This may be solved in six pieces, some of which are rotated.