Reflection on Problem Writing

Writing original problems was not something I looked forward to when I heard about the assignment for the first time. To me, solving conceptual questions was difficult enough but writing them seemed like a daunting task. I admit that I spent a long time on the first original problems we were assigned. For several days I couldnt even start writing a problem because I let myself get overwhelmed by the assignment. I didn't know where to begin. Before coming to IMSA math for me was black and white. For every problem there was one answer. IMSA has changed that for me because there are so many different ways to approach one problem now, for example I could use logarithmic differentiation to find a derivative or I could use the product rule to get there same answer. Original problems seemed like another way of doing math differently and I wasnt excited about it. I was moving even further away from my mind set of math being black and white. I had to be creative when I thought of original problems and being creative was something I never associated with math before.

Throughout this semester I had to write 6 original problems. The first original problems were hard because it was the first time I had to tackle an assignment like this. Somehow I came up with the problems simply by looking through the book and finding an interactive tutorial online; they gave me my inspiration for the problems I wrote. In my first reflection I wrote, I realized it is a lot easier to write problems when you understand the material. And in my graded draft in pencil, Problem writing is challenging! was written after my reflection. As a look over all three original problem assignments I find myself still thinking that problem writing is challenging and also that all of my problems are about material that I understood. On one hand it doesnt seem like my problems have changed much, but on the other hand I feel like I have actually grown as a problem writer. I have more of an idea where to start looking for inspiration. I look at problems I got wrong on Fundays and try to create problems that reflect similar concepts. I found that I like to make graphs for my original problems because it is a lot easier to explain the solution when I have a visual element I can refer to and look at. These patterns are something that can be seen just by looking at my original problems. But ultimately I think being more confident and comfortable with creating my own conceptual questions is the biggest sign of my growth as a problem-writer.

Completing an assignment is one thing, but did writing these problems valuable to me? Honestly, the first two were not valuable to me but the final original problems involving the multiple choice answers was. The first two didnt seem that valuable to me because I could do it in one sitting. I couldnt come up with the problem in one try but after I did I could write the answer and the reflection and be done because there was only one answer. I only had to think about the problem once to solve it. I didnt take the time to thoroughly think about the answer because I only needed to provide one answer.

For the multiple choice original problem I had to think about different ways to solve each problem. I had to think about all of the mistakes I have made or would have made. It forced me to reflect on the subject I was writing about. In m case the problem were about the chain rule and implicit differentiation. It wasnt as easy as I thought it would be to write wrong answers. A lot of the mistakes I ended up making were algebra mistakes and I couldnt write that for four wrong choices. I had to think about the mistakes I had made when I first started learning implicit differentiation or chain rule. I didnt make up all of the answers in one day either. Also, working with a partner made me reflect on my own work. He was able to check my work and show me what I was doing wrong and I could see how other people went about writing original problems. It was also very helpful to have someone I could bounce ideas off of. Overall I feel that the multiple choice original problems helped the most because it required me to think of my own mistakes.

Good work! Let's see an example for Problem #2. A –

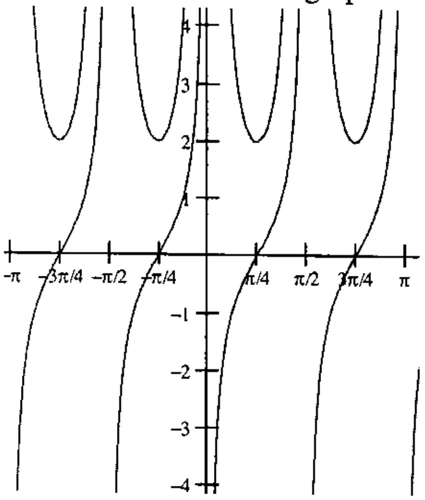
Christina Cheng

Motivation: I found an interactive tutorial on the derivatives of a tangent so I decided to make a question based off of the derivative of the tangent.

Problem Statement: Draw the graph of tan $(2x+\pi/2)$ and determine its derivative. Consider the following questions. How does the concavity of the function effect the derivative? How do the asymptotes effect both graphs? Can you still make a sign chart when there are asymptotes present?

This is good in that you bring in the question of asymptotes.

Problem Solution: The graph and the derivative would look like this



The graph of tan $(2x+\pi/2)$ changes concavity at $x= \pi/2 + \pi/2k$ when k is all integers therefore there will be a local minimum or maximum in the derivative. Even though there are asymptotes you can still create a sign chart and that sign chart will help you determine whether or not the extrema of the derivative will be minimum or maximum points. The asymptotes of the graph tan $(2x+\pi/2)$ will remain the same for the derivative because there are no points at the asymptote so there is no way there can possibly be a slope therefore the derivative also has asymptotes at $x=\pi/2k$ when k is all integers.

Reflection: As a writer I found that it is more difficult to combine math (trigonometry) that I have learned at different periods of time. Even though I am supposed to know trig I still had some difficulty writing the answer for the problem because of the trig because I had to remember about asymptotes and the shift of the graph and then relate the graph to material I learned in BC. I learned that I need to review trig to be able to problems such as $f(x)=\cos(x)\sin(2x)$ but that once you draw the graph it is relatively simple to solve the problem.

Christina Cheng

Motivation: I was flipping through chapter one section four of the book and Figure 3 helped me create my problem because I saw the tangent lines touching the graph of a function.

Problem Statement:

Given the equations of the 3 lines, $y=1x+5$, $y=-(1/2)x-1$, and $y=-x+6$, is it possible to create a graph and also find the derivative of the created graph? If so draw a graph and its derivative.

You should be specific : what kind of graph do you mean.

Problem Solution:

Yes, it is possible to draw a graph but there are many possibilities for the graph. When drawing the graph you would only have to make sure that each line touches the graph exactly once, so then the three lines will be tangent lines of the graph. Once you know what your graph looks like it isn't too difficult to draw an approximation of what the derivative would look like. You would take the slopes of the tangent lines and plot them, it might be necessary to use more than the 3 tangent lines given in the problem. After plotting the points you should keep in mind that when the function has a min or a max then the derivative will have a zero and also when the original graph has an inflection point the derivative will have a local minimum or maximum point. Not only can you use the min and max points to help create your graph but also the fact that when the function is concave up the derivative graph will be increasing and if the function graph is concave down then the derivative graph will be decreasing. Lastly you should remember that when the function graph is increasing then the derivative graph is positive and if the function graph is decreasing then the derivative graph is negative.

OK - but show me an example of what you have in mind!

Reflection: I chose to write this particular problem because at the beginning of BC1 it was difficult for me to grasp the concept that the slopes of the tangent lines helped to determine the derivative. Now that I think about it the concept is very simple, but at the time it seemed so confusing. So I took the problem of drawing a derivative graph from a function and added a step of drawing the first graph from the tangent lines. As a problem writer I realized it is a lot easier to write problems when you understand the material and not only material that you know well but material that you originally had difficulty with because I could just think of some of the key things about that concept and try to work it into the problem. However it is not easy to think of the question because a lot of the time I found myself writing problems that were essentially the same as ones I had done previously. Problem writing is challenging!

Your second problem is good, but the first is essentially a restatement of Theorem 8.

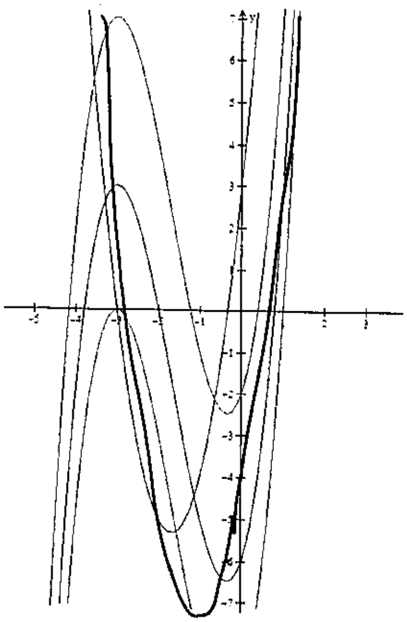
B+

Christina Cheng

Motivation: I was looking through my notes on sections 2.4 through 2.6 and I saw some of the equations had +C attached to the end of them and I remember ignoring those when I was reviewing because I didn't really think that they were important but when I searched for some example anti-derivative problems online there were a lot of equations with constants attached to them, so I decided to see why the constant was important.

Problem Statement: Is the following statement true or false? Please explain your reasoning and support your reasoning with an example. If *f(x)* is a function, than *f(x)* has only one antiderivative.

Problem Solution: The statement if *f(x)* is a function, than it only has one anti-derivative is always false! Adding a constant to an antiderivative only changes its position on the y-axis and not the shape of the graph. There are an infinite amount of points on the y-axis so there will be an infinite amount of antiderivatives possible for a function *f(x)*. Why is this? In order to take the derivative of a function one looks at the slope of the tangent line to figure out the derivative, so since the derivative of the anti-derivative of *f(x)* we would look at the slope of the antiderivative. If the shape of the anti-derivative is always the same even when a constant (+C) is added than the slope of the tangent line will not change and the derivative will remain the same, therefore there is not only one anti-derivative for a function of *f(x)*.



For example, $3x^2 + 10x + 3$ is graphed in red and the antiderivative is $x^3 + 5x^2 + 3x + C$ when you use the power rule to find the antiderivative. The +C is the constant that translates the graph of the anti-derivative up and down but as one can tell the graphs remain the same so the slopes of the tangent lines are the same. When using winplot you can hit derive after graphing the antiderivative and what you will get is the graph represented in red, the derivative of the antiderivative which was the function you began with ($3x^2 + 10x + 3$). It is easier to see this problem in winplot because one can tell that all of the derivatives, of the antiderivatives with different constants, have the same graph.

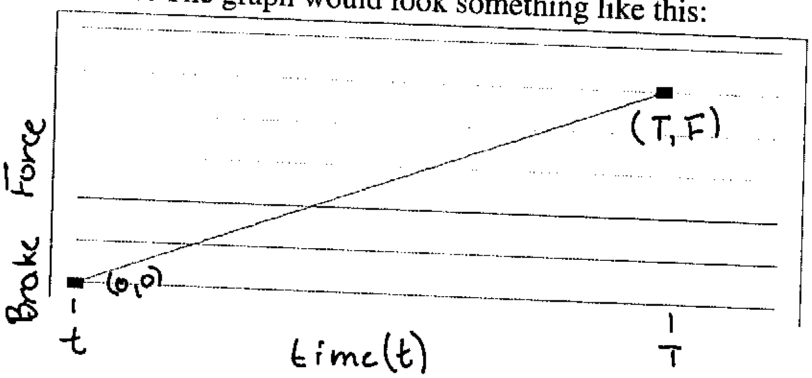
Reflection: I learned that the constant of an anti-derivative does not affect the graph of its derivative. From this I learned that graphing equations makes it easier to understand concepts rather than just reading about it or hearing about it because actually watching the derivatives of my graphs have the same line solidified the concept in my mind. I found that when it comes to derivatives I am more of a visual learner.

Christina Cheng

Motivation: I looked at some of the practice problems in chapter 2.4 and realized that I never really understood how to apply derivatives to the real world such as in question 24 and one of the fun day questions, so I thought I would try writing my own application problem because at times it seems like I will never have to use calculus in the real world. I looked up some examples online and found an equation describing airplane breaks and felt it was an interesting application of derivatives that I had not seen before so I modeled my question off of the website (www.kidwaresoftware.com) question.

Problem Statement: Suppose a pilot is landing at O'hare international airport in Chicago. We assume the pilot applies the brakes at time zero ($t = 0$). Up to this time, the braking force is zero. It takes T seconds to reach full braking force F, and after this time, the full braking force is maintained. How would we describe the braking force as it transitions from zero to F?

Problem Solution: We could treat this question as if it were asking us to graph the plane's breaking force. In this case we would plot (0,0) because at time 0 there was a force of 0, we would also plot (T,F) because T would be our final time that is required to reach the full breaking force F. The graph would look something like this:



The function of the breaking force over time f(t) would be $f(t)= F\,(t/T)$ and we know that the time starts at zero and ends at T the interval of t would be $[0,T]$. We learned that a straight line like this, which would have a sharp corner after the point (T,F) as the force remained constant cannot happen in real life because such a change would not be so sudden, also at the sharp corners of this curve a derivative would not exist. In real life the line would be a smooth curve with existing derivatives because everything happens at a rate, which is what the derivative is. The best way to describe the braking force of the plane would be to create a function and the derivative that would fit the requirements that are implied in the problem. $f(0)=0$, $f(T)=F$, $f'(0)=0$ (we know this because before the brake is used the force of the brake is constant which has a zero derivative) and $f'(T)=0$ (we know this because after the brake reaches its full force it will remain constant until the plane stops so the derivative of that constant brake force is also 0).

Reflection: The example question was very difficult to understand at first but I read it multiple times and I was able to identify pieces of information that I knew. This question helps me to understand the practical uses of derivatives in the real world, someone such as an engineer or a plane designer might do a calculation like this while designing new brakes for airplanes.

Interesting problem! Can you sketch a possible function for the braking force?

Seth Zelman, Christina Cheng

Early

4/29/11

ORIGINAL PROBLEMS

A −

Original Problem 1

Motivation

We chose to do a problem involving the chain rule, product rule, and quotient rule because we recently had a FunDay involving these rules. We looked at problem 8 from the FunDay and based our original problem off of that. We also wrote this problem because we both were having trouble understanding these types of problems and thought that by creating our own, we would gain better understanding of the concepts.

Problem

Which of the following is $\frac{d}{dx}\frac{g(f(x))(h(x)^4)}{k(x)}$?

As I mentioned before, this is more at a skills level than a conceptual level.

(A) $\frac{[g'(f(x))f'(x)h(x)^4 + 4g(f(x))h(x)^3h'(x)][k(x)] - [g(f(x))h(x)^4][k'(x)]}{k(x)^2}$

(B) $\frac{[g'(f(x))f'(x)h(x)^4 + 4g(f(x))h(x)^3h'(x)][k(x)] + [g(f(x))h(x)^4][k'(x)]}{k(x)^2}$

(C) $\frac{[g'(f(x))h(x)^4 + 4g(f(x))h(x)^3][k(x)] - [g(f(x))h(x)^4][k'(x)]}{k(x)^2}$

(D) $\frac{[g(f(x))h(x)^4][k'(x)] - [g'(f(x))f'(x)h(x)^4 + 4g(f(x))h(x)^3h'(x)][k(x)]}{k(x)^2}$

(E) $\frac{[4g(f(x))h(x)^3][k(x)] - [g(f(x))h(x)^4][k'(x)]}{k(x)^2}$

Solution

The correct answer is:

$$(A)\frac{[g'(f(x))f'(x)h(x)^4 + 4g(f(x))h(x)^3h'(x)][k(x)] - [g(f(x))h(x)^4][k'(x)]}{k(x)^2}$$

Using the chain rule and then the product rule, the derivative of the numerator can be found. First, use the the chain rule for each combination of functions in the numerator to get

$$[g'(f(x))f'(x)][4h(x)^3h'(x)].$$

careful! what are you saying this is?

“1”

The product rule can then be used in the numerator to find the derivative

$$g'(f(x))f'(x)h(x)^4 + 4g(f(x))h(x)^3h'(x).$$

Lastly, use the quotient rule to find the derivative of the entire function. If done correctly, the final result will be

$$(A)\frac{[g'(f(x))f'(x)h(x)^4 + 4g(f(x))h(x)^3h'(x)][k(x)] - [g(f(x))h(x)^4][k'(x)]}{k(x)^2}$$

Discussion

The correct answer is (A).

When using the quotient rule, if the student accidentally added instead of subtracted, they would choose answer (B).

Answer (C) could be chosen if a student incorrectly used the chain rule. If they found the ~~derivative~~ of $g(f(x))$ to be $g'(f(x))$ and the derivative of $\cancel{h(x)^4}$ ~~to~~ be $4h(x)^3$, ignoring the extra f'(x) that is present with the first derivative and the h'(x) that is present with the next derivative, they would have wrong answers for those ~~derivatives~~. Then, after using the product rule, they would get the derivative of the entire numerator to be

$$g'(f(x))h(x)^4 + 4g(f(x))h(x)^3.$$

After using the quotient rule, they would get answer (C), which is incorrect.

If the student mixed up the quotient rule and used the equation

$$\frac{d'(x)n(x) - n'(x)d(x)}{(d(x))^2}$$

instead of

$$\frac{d(x)n'(x) - n(x)d'(x)}{(d(x))^2},$$

they would select (D) as their answer.

Answer (E) could be found if a student saw the numerator of the original function $g(f(x))h(x)^4$ as one term and first used the power rule to derive the numerator $4g(f(x))h(x)^3$. After finding this derivative, if they used the quotient rule they would get answer (E)

$$\frac{[4g(f(x))h(x)^3][k(x)] - [g(f(x))h(x)^4][k'(x)]}{k(x)^2}$$

Reflection

Through the process of writing this original problem we learned that knowing the different rules is imperative to answering these types of questions. If you get one rule wrong, whether it is the quotient rule, product rule, or chain rule, it will alter your answer. Not only is it important to know what the rules are, but we also learned that you have to know when to apply them. It's easy to forget to apply one of the rules if more than one is required for the problem. Having to come up with four different ways that a student could incorrectly solve the problem allowed us to reflect upon our own work. It helped us to realize some possible mistakes that we could make, so that in the future we will be less likely to make those errors.

Original Problem 2

Well-designed problem!

Motivation

We decided to base our problem off of implicit differentiation because we had just learned it in class and were having difficulties understanding the process. We thought that this problem was a great way to improve skills with implicit differentiation, as well as to practice basic algebra by analyzing different types of mistakes. This problem also recalls basic algebra skills by requiring the use of the point-slope equation, as well as knowing that the slopes of perpendicular lines are opposite reciprocals.

Problem

For the implicit function $x^3 + 2xy + y^2 = 1$, which of the following is the equation of the line perpendicular to the tangent line at the point $(1, -2)$?

(A) $y = -\frac{1}{2}x - \frac{3}{2}$

(B) $y = 2x - 4$

(C) $y = -\frac{2}{5} - \frac{8}{5}$ where is x ?

(D) $y = \frac{5}{2} - \frac{9}{2}$

(E) $y = x + 1$

Solution

The correct answer is (B) $y = 2x - 4$. First, differentiate the function $x^3 + 2xy + y^2 = 1$. To do this, derive both sides to get

differentiate

$$3x^2 + 2y(x) + 2xy'(x) + 2y(x)y'(x) = 0.$$

This can be rearranged to get

$$y'(x) = \frac{-(3x^2 + 2y)}{(2x + 2y)}.$$

After substituting the point $(1, -2)$ into the derivative function

$$\frac{-(3(1)^2 + 2(-2))}{(2(1) + 2(-2))},$$

the result is $-\frac{1}{2}$. This gives the slope of the tangent line. We know that the slope of the line perpendicular to the tangent line is the negative reciprocal of the slope of the tangent line, so it would be 2. Knowing this, we can use the point-slope equation with the points given and the calculated slope

$$y - (-2) = 2(x - 1).$$

This yields the equation of the line $y = 2x - 4$.

Discussion

The correct answer is (B).

A student could get answer (A) $y = -\frac{1}{2}x - \frac{3}{2}$ if they used the calculated slope from the derivative instead of taking the opposite reciprocal. The slope from the derivative is the slope of the line tangent to the curve at the point $(1, -2)$; to get the slope of the line perpendicular to that line, the opposite reciprocal must be used.

If a student forgot to use the chain rule for the third term of the original function (y^2), they would get the equation of the derivative to be

$$y'(x) = \frac{-(3x^2 + 4y)}{(2x)}.$$

Then, when substituting the point $(1, -2)$ into the function, they would get the answer to be $\frac{5}{2}$. After taking the opposite reciprocal of the slope and using the point-slope equation, their equation for the line would be (C) $y = -\frac{2}{5}x - \frac{8}{5}$ and this is incorrect.

To get answer (D) $y = \frac{5}{2}x - \frac{9}{2}$, a student could have made the same mistake as in answer (C), but forgot to take the opposite reciprocal of the slope and used the slope of the tangent line instead.

If a student forgot to take the derivative of the right side of the original implicit function, they would get their derivative to be

$$y'(x) = \frac{1 - 3x^2 - 2y}{2x + 2y}.$$

Good choice of distractors!

After substituting the point $(1, -2)$ into this incorrect derivative function, they would get a slope of -1. After reciprocating it and using its opposite value in the point-slope equation with the original point, they would get an equation of (E) $y = x + 1$.

Reflection

Originally, the choices for this problem included the words decreasing and increasing. However, we learned that we could not use these terms for the graph. As a result we changed our problem to include another step, finding the equation of a line that was perpendicular to the tangent line at the point (1,-2). This added another aspect to our problem and also eliminated the incorrect vocabulary. By writing this problem, we practiced implicit differentiation. The practice has helped us feel more comfortable doing math problems that involve implicit differentiation.