Original Problem Reflection

Over the course of the semester, I have been writing original problems to correlate with class topics. At first, the idea of writing original problems was a little daunting because I had never written my own problems before. I had no idea where to begin or how to think of problems. However, as time went on, I was able to maintain a firm grasp on the process of problem writing.

Looking back on my first set of original problems, I realized my motivation was mainly topics in the book. The problems were simple because I did not have a lot of background knowledge of calculus at the beginning of the semester. A lot of the problems were skill related, and not too difficult. However, looking at my second and third set of original problems, I found my problems often sought to expand on knowledge, instead of just applying it. As a result, I was able to write more difficult problems which helped me understand and anticipate FunDay problems.

Furthermore, instead of focusing on one topic per question, I incorporated multiple topics in my more recent problems. For example, in my skills problem on the second original problems assignment, I incorporated hyperbolic trigonometry with the definition of a derivative. Combining topics together helps me understand them in a more efficient manner.

I became more confident and comfortable with my problem writing skills over the course of the semester. Part of this was because I had practice throughout the semester. With lots of practice, I was able to hone my skills as a problem writer. Using technology and thinking of problems became easier. Another reason for my confidence was because we had many

FunDays. On the FunDays, I was able to see which types of problems were conceptual, and which types of problems were skill related. This allowed me to assess my own problems easily, and write them consistently.

I believe problem writing is a valuable assignment because it allowed me to thoroughly think through and understand concepts without having to do a vast number of book problems. Although it takes roughly the same amount of time, I was able to completely understand a topic because I had to write a detailed solution to each problem. When I do book problems, I find the answer and move on to the next problem. However, when I write original problems, I double-check my solution is correct and thoughtfully explained.

Furthermore, original problem assignments proved to be very valuable because I learned how to use technology. For the first two assignments, I used Microsoft Word. I had not known about the function option on Microsoft Word. As a result, it took a bit of time to learn how to use Microsoft Word efficiently. Inserting all the symbols and equations was tedious. On my third assignment, we learned how to use LaTeX. It was slightly difficult to grasp, but it was more efficient than Microsoft Word. It's also simple to use for inputting special mathematic equations and symbols. Using LaTeXwill be useful in the future when I want to write problems.

Writing original problems proved to be very valuable because it allowed me to connect different concepts from the semester together. At the beginning of the semester, I was taught different ideas without a firm understanding of why they were true. However, as time went on, I was able to connect newly learned knowledge to the earlier topics and confirm what I had previously learned. Original problems were also valuable because they helped me anticipate FunDay problems. I remember being worried about FunDays because of the conceptual problems. However, after writing a couple conceptual problems, I found FunDays to be easier. I was able to think clearly about concepts and thoroughly explain my solutions.

Overall, original problems proved to be very valuable to my understanding of calculus. I was able to assess my weaknesses in calculus through the writing of original problems. The original problems assignments also helped me become an efficient and thorough problem writer. This assignment should be used in other math classes as well, such as the MIs, because it helps prepare students for assessments and also helps them consider their own weaknesses.

Diana Xu – Matsko

A

Original Problem #1 – Chapter 1 - Skills

1. In section 1.4, the calculus book explains the speed limit law. Also, on the last FunDay exam, a conceptual problem focused on the application of the speed limit law and what could be said about the function's value at a certain point. As a result, I decided to write a similar problem where students must use the speed limit law to estimate a function's value at a certain point. I also wanted to include some review of algebra when dealing with inequalities.
2. Suppose that $7 \leq f'(x) \leq 13$ for all x. Find the range of possible values for $-f(3)$ if $f(-6) = 4$. In complete sentences, explain how you arrived at your solution.
3. Since we know the range of values for $f'(x)$, we can use the speed limit law to find out a range of values for specific points on the $f(x)$ graph. First, we must set up the inequality to be $7(3-(-6)) \leq f(3) - f(-6) \leq 13(3+6)$. The equation can be rewritten as $64 \leq f(3) - 4 \leq 117$. Simplifying the equation results in $68 \leq f(3) \leq 121$. Since the problem is asking for a range of possible values for $-f(3)$, the whole inequality must be multiplied by -1. By doing this, we find the solution to be $-68 \geq -f(3) \geq -121$.
4. I learned I can take fairly simple concepts and develop them into more difficult skill problems. I observed I like to try and find different ways to expand the application of basic facts found in the book. In this case, instead of using a one sided inequality for the speed limit law, I tried using a two sided inequality. It was easier for me to write the solution to this problem, compared to the previous problem, because I had all the knowledge needed to solve it. I also like to incorporate small algebra tricks to remind myself of basic algebra rules.

Or perhaps: begin with $7 \leq -f'(x) \leq 13$, and ask for $f(3)$.

Diana Xu – Matsko

Original Problem #2 – Chapter 1 – Conceptual

1. Example 4 of section 1.6 states the derivative of sin (x) is cos (x). I knew this to be a fact, but it was vague as to why it was true. Also, on the last FunDay exam, there was a conceptual problem that used trigonometric functions. As a result, I decided to write a problem asking the student to apply what they knew about trigonometry and finding derivatives.

2. Prove that the derivative of $\sin(x)$ is $\cos(x)$.
3. Using the definition of a derivative and difference quotients we can prove the derivative of $\sin(x)$ is $\cos(x)$. First, we must set up the difference quotient as $\frac{\sin(x+h)-\sin(x)}{h}$. From here we can apply the trigonometric theorem of the difference of two sines to the numerator to get $\frac{2\cos\left(\left(\frac{1}{2}\right)(x+h+x)\right)\sin\left(\left(\frac{1}{2}\right)(x+h-x)\right)}{h}$. This equation can be rewritten as $\frac{2\cos\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$, which can be further rewritten as $\frac{\cos\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$. The fraction can be split up into $\cos\left(x+\frac{h}{2}\right)\cdot\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$. Since $\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta}=1$, I can substitute 1 in for $\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$. From here, I can simplify the previous fraction to $\cos\left(x+\frac{h}{2}\right)$. To find the limit as h goes to 0, we substitute 0 in for h. Therefore, we find $\frac{dy}{dx}\sin(x) = \cos(x)$.
4. I learned it is difficult to come up with original problems, especially when I do not have an idea on how to solve them. This problem was difficult for me because I had to research if $\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta}=1$ was true. As a problem writer, I noticed I like to write problems I found myself wondering about. In this case, I was curious about the method of proving the derivative of $\sin(x)$ is $\cos(x)$. At times, it can be difficult to solve my problems because I do not have enough calculus knowledge. In this case, I did not know $\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta}=1$. Without this knowledge, I could not have solved the problem. I noticed this problem was much more difficult to solve than the first problem. It took more thought and knowledge to solve.

Good! An original way of finding the derivative of sin x. However, there is the detail of substituting for h at two different times. This does not always work.

Diana Xu - Matsko

B+

Original Problem #3 – Chapter 2 – Skills

1. On the last FunDay exam over chapters 2.1, 2.2, and 2.3, there was a problem asking us to recognize and use the limit definition of the derivative to find the limit as x approached $\pi/2$ of $\frac{\cos(x)}{x-\pi/2}$. However, I was unable to solve the problem correctly because I did not recognize it to be an application of the limit definition of the derivative. Therefore, I decided to write a problem asking students to apply the limit definition of a derivative. To add to the difficulty level, I decided to incorporate hyperbolic trigonometric functions in the problem. Also, in section 2.1, problems 48-51 asked us to use the limit definition of a derivative to evaluate the limits. I was confused about how to solve those problems, so I figured it would be a good idea to write an original problem focusing on the limit definition of a derivative.
2. Evaluate $\lim_{x\to 0}\frac{\cosh(x)-1}{x}$.
3. To find this limit, we must use the limit definition of a derivative. The derivative of a function at $x = a$, denoted by $f'(a)$, is defined by the limit equation $f'(a) = \lim_{h\to 0}\frac{f(x+a)-f(a)}{x}$. In this problem, it is determined that $a = 0$. Therefore, the limit definition of a derivative states the limit of $\frac{\cosh(x)-1}{x}$ is equal to $\cosh'(a)$. We know the derivative of $\cosh(a)$ is $\sinh(a)$. As a result, we can find the limit by substituting 0 for a. This can be rewritten as $\sinh(0)$, which is equal to 0. Therefore $\lim_{x\to 0}\frac{\cosh(x)-1}{x} = 0$.
4. As we continue to progress through BC calculus, I find myself confused more often on certain topics. I attempt to do all the book problems, but I still have uncertainties for a couple of ideas. One of these is recognizing when to use the limit definition of a derivative. After writing this problem, I realized how it is sometimes simple to recognize when to apply to limit definition of a derivative. I noticed I like to combine earlier topics with recent topics when I write my problems. For example, in this problem I combined the limit definition of a derivative with hyperbolic trigonometric functions. This motivates me to continue to review previous topics, while adding onto my knowledge at the same time. I realized it is important to connect important ideas in calculus together to test my knowledge of both ideas. Writing original problems is slowly becoming simpler because I can find my weak points and write problems about them.

This is a good problem to incorporate hyperbolic trigonometry

Diana Xu - Matsko

Original Problem #4 – Chapter 2 – Conceptual

1. In the chapter 2 summary problems, I spent a great deal of time on problem #59 because I was confused about how to approach the problem. It asked me to prove using calculus that all values of the function were positive ~~for every x~~. I looked at all the relationships between a function, its derivative, double derivative, and anti-derivative. I knew if a function was always positive, then its anti-derivative would always be increasing. However, I did not know how to progress using this idea. Eventually, I decided to find the minimum value of the function to see if it was negative. If the minimum value of the function was negative, the function was not positive for every x. On the contrary, if the minimum value of the function was positive, the function must be positive for every x. I decided to create an original problem in which this method of proof can be utilized.

anti ??

2. Is the value of $f(x) = -4x^2 + 6x - 1$ negative for every x? Explain your answer using calculus. DO NOT graph the function to explain your answer.

3. The best point of representation for this problem is the maximum value of $f(x)$. If the maximum value of $f(x)$ is negative, then the function is negative for every x. However, if the maximum value of $f(x)$ is positive, then the function is not negative for every x. Using calculus, we can find the maximum value of this function by finding the zero of the derivative, and substituting it into $f(x)$. The derivative of $f(x)$ is $f'(x) = -8x + 6$. Setting the derivative equal to 0 allows us to find what x value the maximum value occurs on for $f(x)$. This can be written as $0 = -8x + 6$. Solving for x results in $x = \frac{3}{4}$. That means at the point $x = \frac{3}{4}$, the function $f(x)$ reaches its maximum value. Substituting $\frac{3}{4}$ for x results in $f\left(\frac{3}{4}\right) = -4\left(\frac{3}{4}\right)^2 + 6\left(\frac{3}{4}\right) - 1$. Solving the equation results in $f\left(\frac{3}{4}\right) = \frac{5}{4}$. Since the maximum value of $f(x)$ is positive, the function is not negative for every x.

How do you know this is a maximum and not a minimum? You need to look at the second derivative.

4. When I am writing original problems, I find it easier to write it on paper before I type it on the computer. It helps me organize my thinking and generally prevents any mistakes in my solution. I will continue to do this throughout the semester. Once again, I realized I tend to write problems based on my weaknesses in BC calculus. I figure if it is difficult for me to think through a type of problem, then it could be difficult for other people as well. It was more difficult to write the solution to this problem, compared to the previous problem. This is because it is harder for me to pinpoint weaknesses in understanding concepts than to find my weaknesses in basic skills.

Still a good idea....

Yes - good approach. But I think you could look for a more interesting function - maybe not a parabola. It is easy to see that $f(1) = 1$, and so f is not always negative good discussions, though the 1st problem is not as conceptual as the secon

The stories make the word problems more entertaining!

Anti-Derivatives and Integrals

A–

Problem 1

Motivation: Earlier in the semester, we found displacement by using the area under a velocity graph. However, since all the graphs consisted of linear functions, it was easy to determine the area underneath the function. To expand on this, we decided to create a problem in which we needed to find displacement by finding the area under a non-linear velocity graph. To do this, we utilized our knowledge of trigonometric and logarithmic derivatives and anti-derivatives to find the displacement from a given velocity function. We also looked on how to compute integrals to help solve our problem.

Problem: An ant is making its way back to an ant hill. The ant hill is 5 meters away from where he began his journey 3 seconds ago. If it is currently time 0, and his velocity in meters per second is expressed by the function

$$\cos\left(\frac{\pi x}{2}\right) + \frac{1}{x+4},$$

Shouldn't this be t?

how much further does he have to walk to reach the hill?

(A) $\left(\frac{5\pi - 2}{\pi}\right) - \ln(4)$ meters

(B) $\left(\frac{5\pi + 2}{\pi}\right) - \ln(4)$ meters

(C) $\ln(4) - \frac{2}{\pi}$ meters

(D) $\frac{2}{\pi} - \ln(4)$ meters

(E) $\ln(4) - 1$ meters

Solution: The correct answer is (B). First, we must find the distance the ant has already traveled in 3 seconds. We must first find the anti-derivative of our velocity function. The anti-derivative of $\cos\left(\frac{\pi x}{2}\right)$ is $\left(\frac{2}{\pi} \times \sin\left(\frac{\pi x}{2}\right)\right)$. The anti-derivative of $\frac{1}{x+4}$ is $\ln(x+4)$. To find the distance the ant already traveled in 3 seconds, we must compute

$$\int_{-3}^{0} \left(\cos\left(\frac{\pi x}{2}\right) + \frac{1}{x+4}\right) dx = \frac{2}{\pi}\sin\left(\frac{\pi x}{2}\right) + \ln(x+4) \Big|_{-3}^{0}$$

The bounds of the integral are substituted for x to find the area under the velocity function. As a result, we find the integral to be $\ln(4) - \dfrac{2\sin\left(\dfrac{-3\pi}{2}\right)}{\pi}$. This can be rewritten as $\ln(4) - \dfrac{2}{\pi}$. This value represents the distance the ant already traveled in 3 seconds. To find the remaining distance the ant must travel, the value must be subtracted from 5 meters. As a result, we find the ant must travel another $\left(\dfrac{5\pi+2}{\pi}\right) - \ln(4)$ meters.

DISCUSSION: We know the correct answer is (B). If a student forgot to use the distributive property when subtracting from 5 or if the student added while computing the integral, they would choose (A). If a student forgot to subtract the integral from 5 meters, they would choose (C). If a student flipped the bounds of the integral, they would choose (D). If a student found the anti-derivative of $\cos\left(\dfrac{\pi x}{2}\right)$ to be $\sin\left(\dfrac{\pi x}{2}\right)$, they would choose (E).

REFLECTION: Originally, this problem had just been about finding the integral of a function. However, we thought it would be too simple. Instead, we decided to make it a word problem because students often get confused about word problems. Usually students miss important steps in word problems because there are a lot of conditions in the problem. We thought it was interesting to combine concepts we learned at the beginning of the semester with anti-derivatives we recently learned. It also reminded us of the significance of the area under a function. This was our first time creating multiple choice questions. It was not too difficult thinking of choices because, as students, we know what kind of mistakes we make. This was also our first time using LaTeX. Using LaTeXwas a little confusing because we often could not find our mistakes in coding. However, it was nice to have built-in functions because we did not have to worry too much about the different formatting of functions and variables.

PROBLEM 2

MOTIVATION: The motivation for this problem came from Paul Cladek. Previously, we were thinking about doing an optimization problem. The problem was not conceptually hard, just tedious. While all the information is given in the above problem, everything is not explicitly said. Paul gave us the idea to create an anti-differentiation problem giving minimal information. We both had weaknesses during the anti-derivative unit and the corresponding FunDay, specifically with finding constants. Further, we thought it would be interesting to write a word problem because students often get confused when trying to solve them.

PROBLEM: At the Matsko 500, two race cars are competing for first place. Dr. Matsko's godson, Risun, and his colleague, Dr. Prince, both start their last lap at the same time, but Risun is going 60 meters per second and Dr. Prince is going 80 meters per second. Risuns acceleration is expressed by the function $a(t) = t^2 - 3t + 4$, and Dr. Princes acceleration is expressed by the function $a(t) = 2t^2 - 10t + 10$. The last lap is 300 meters long. Find which driver will receive the first place trophy and give the displacement function.

(A) *Risun*, $\frac{1}{12}t^4 - \frac{1}{2}t^3 + 2t^2 + 60t$

(B) *Dr.Prince*, $\frac{1}{6}t^4 - \frac{5}{3}t^3 + 5t^2 + 80t$

(C) *Dr.Prince*, $\frac{2}{3}t^3 - 5t^2 + 10t + 80$

(D) *Risun*, $\frac{1}{4}t^3 - \frac{3}{2}t^2 + 4t + 60$

(E) *Dr.Prince*, $\frac{2}{3}t^3 - 5t^2 + 10t$

why in math mode?

SOLUTION: Sadly, Dr. Matsko's godson did not win the trophy. Dr. Prince won by less than a second. Choice (B) is the correct answer. From the given acceleration functions, the displacements functions needed to be found. If we anti-differentiate Dr. Prince's acceleration, we find the velocity, which is

$$v(t) = \frac{2}{3}t^3 - 5t^2 + 10t + C.$$

The constant, C, can be found by substituting the initial data, 0 seconds in for t, and 80 meters per second for $v(t)$. To solve this problem, you must define the beginning of the final lap as zero seconds. The initial speed of Dr. Prince is given in the problem, which is 80 meters per second. Now that we found the velocity function, we must again anti-differentiate to find the displacement function. The displacement function is

$$s(t) = \frac{1}{6}t^4 - \frac{5}{3}t^3 + 5t^2 + 80t + C.$$

Again, this constant can be found by substituting in 0 for t and $s(t)$. To solve for which car will pass the finish line first, you need to substitute 300 for $s(t)$ and solve for x. You find that Dr. Prince passed the finish line first, and his displacement function is

$$s(t) = \frac{1}{6}t^4 - \frac{5}{3}t^3 + 5t^2 + 80t.$$

DISCUSSION: We know choice (B) is the correct answer. Choice (A) is incorrect because when finding the time, Risun passed the finish line less than a second late. Solving for x shows that Dr. Prince passed the finish line first. Choices (C) and (D) are incorrect because those functions are the velocity functions, not the displacement functions. A student who chose either choice (C) and (D) only anti-differentiated the accelerations once, finding the velocity function. To find the correct answer, a student must anti-differentiate the acceleration function twice to find the displacement function. A student would choose choice (E) if they only found the velocity function, instead of finding the acceleration function.

displacement

Isn't this choice (C)?

REFLECTION: We really enjoyed making this problem. This problem needed us to both think about what the y and t represented for each function. We both usually get really confused when we anti-differentiate, and this problem helped us hone my skills with anti-differentiation. Because it was a multiple choice question, we were able to see what mistakes we are prone to making. I, Risun, recently just understood when you anti-differentiate you need to add a constant. This problem reemphasized the importance of this concept. Overall, this problem was a lot of fun to make. Risun's take on LaTeX: I really enjoyed using LaTeX. It made inserting equations easier and more efficient. I hated using Microsoft Word for original problems because it was extremely tedious to insert each symbol. Diana's take on LaTeX: It was a little confusing, but after awhile, it was pretty simple to use. Most of the mistakes we made were due to small errors in code that were easily fixable.