Summary Comments on Documents Received

Below are some introductory comments on the materials sent to me so far. These comments indicate directions I might take in workshops during my visit.

The comments are based on translations (by one of my Thai students) of the lesson on estimation as well as the activity for learning about the base 3 number system. My student did not feel he had the background to translate the discussion of the real number system, but by looking at the mathematical symbols, I think I can follow the basic outline of the mathematical development.

Adventure in Ket-Kham City

I enjoyed reading this activity. I think it is a good exercise in problem-solving, and equally importantly, involves students in a "hands-on" activity. I believe that students do learn better when they are actively engaged in learning (although this does not always mean physical activity – students may be engaged in an interesting paper-and-pencil task).

I wondered about the remark at the end not to mention the words "Base 3" at any time, even at the end of the investigation. Will there be a discussion of this idea later? I looked through your Standards, and it seemed that Standard M 6.1 was the only one which might fit. It would seem important at some time to discuss the activity in the context of the Base 3 number system.

This activity might lead into another. Suppose, now, that students are given square coins cut out of different colors of paper, and are given three of each. Then ask the students to create problems like the Adventure in Ket-Kham City. Students may share some of their problems with others in the class, or the teacher might choose particularly good problems for the class to do.

If this is successful, perhaps students can be encouraged to think about the relationship between the two activities. In this way, it may be possible for students to discover the idea of a "number system" on their own.

Estimation

There are many good ideas in this set of problems, many of which clearly support students achieving Standard M 1.3. Using examples involving money is a way of introducing mathematics into daily life. It might be a fun activity to make a "grocery store" in the classroom and give the students play money. They might even make up problems for each other.

There may also be other ways to involve estimation in the classroom. Some examples might be:

1. You need to purchase carpeting for your classroom. How many square meters will you need?
2. Suppose your classroom is converted to an aquarium! How many liters of water would you need to fill it?
3. Fill a jar with jelly beans (do you have this candy in Thailand? Any small candy will work). How many jelly beans are in the jar?
4. Fill a container with a large number of small coins (many satang). How much money is in the container?
5. How many students could fit standing up in your classroom if all the desks and furniture were taken out of it?
6. Look out the window at a tall building. How many meters high is it?
7. Take a book in your classroom. Using the same size numbers used in the book, suppose the numbers from 1 to 1,000,000 were written in order in a similar book. How many pages would be in this book?
8. Suppose that all the students in the class stand in a circle so that their arms are stretched out and only their fingertips are touching. What would be the radius of this circle?

Many of these problems involve spatial reasoning as well as some knowledge of geometry. Perhaps students (in small groups) can come up with different estimates and present them to the class. They must justify their estimates. If others make different estimates, why is this? What assumptions were made in making the different estimates? This could provide a very interesting class discussion.

The Real Number System

This is the section I was not able to get translated, but rather followed along with the mathematical symbols. The discussion of the real number system is a good discussion to have; I am interested to know how students respond to this unit. Many ideas here would be too abstract for the average American high school student. If this is taught in all Thai high schools, it is truly an achievement!

§**2.1**: It would be interesting to present the proof from number theory that $\sqrt{2}$ is irrational. Briefly: Suppose that $\sqrt{2} = p/q$, where the fraction p/q is in lowest terms (that is, p and q have no common factor). Then $2q^2 = p^2$. Since $2q^2$ is even, then so must be p^2, and thus p is even. Writing $p = 2r$, we get $2q^2 = 4r^2$, so that $q^2 = 2r^2$. A similar argument shows that q must also be even. But this yields a contradiction: we assumed p and q have no common factors, but conclude that p and q are both even. Thus $\sqrt{2}$ cannot be rational.

§**2.2**: Perhaps include more problems like Exercise 4 (which involves a definition of $\oplus$). I find that one difficulty with this material is motivation: why would one need to learn these various properties? A few examples help provide some contrast.

- Function composition: The function $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$ for all suitable x. This is an associative operation with an identity, but is not commutative. Some functions have inverses, and some do not.
- Matrix multiplication: This is a special case of function composition. Looking at 2×2 matrices provides many examples of the various properties. One interesting example is the fact that two non-zero matrices may have a product which *is* the zero matrix.

All the properties in this section are shared by both the rational numbers and the real numbers. An interesting, though abstract, discussion would be this: What is the difference between the rational numbers and the real numbers in terms of their properties?

§**2.3**: One suggestion I would make for this section is to introduce a graphical component. At IMSA, many of us use a program called Winplot fairly often; it is available free for download at `http://math.exeter.edu/rparris/winplot.html`.

As an example, consider dividing $x^2 - 5x + 7$ by $x - 2$. The result is

$$\frac{x^2 - 5x + 7}{x - 2} = x - 3 + \frac{1}{x - 2}.$$

Looking at a graph of the left expression, one can interpret the quotient as an asymptote to this curve. This is because as $|x|$ gets large, the remainder term gets very small, meaning the

the graph of $\dfrac{x^2-5x+7}{x-2}$ gets nearer to the graph of $x-3$. Many ideas in this illustration may be illustrated graphically.

Also, we frequently have students graph functions to find a first root rather than have them use the rational root test (that is, looking at quotients of factors of a_0 and a_n). This is the subject of many interesting discussions: given that we have easy access to technology, what skills should students be able to demonstrate with pencil and paper alone? There seems to be no easy answer to this question.

§§**2.4** and **2.5**: I find that these ideas are difficult to teach students. You might explain that $\dfrac{1}{x}<2$ does *not* imply that $1<2x$, but some short time later, they make the same mistake again. It seems that these ideas do not "stick." I would be interested to know if these ideas stick with your students, and how you accomplish this.

Again, a graphical approach might help students here – and would provide a check for their work. Let us suppose a student solved the inequality $\dfrac{1}{x}<2$ and got the solution set $(1/2,\infty)$. To check their answer, they might graph $y=\dfrac{1}{x}$ and $y=2$ on the same graph, and see when the graph of $y=\dfrac{1}{x}$ is below the graph of $y=2$. If their answer is correct, this would occur only on the interval $(1/2,\infty)$.

But of course this is not correct; the graph of $y=\dfrac{1}{x}$ is below the graph of $y=2$ on the interval $(-\infty,0)$ as well. Using this approach might help students check their own work and reinforce a difficult concept to grasp.

§§**2.6** and **2.7**: I would again comment on bringing a graphical approach into these sections. Some of the problems (such as Exercise 1 in §**2.7**: $|x-2|<1$) can be easily solved using graphical methods. Using graphs gives the more visual learner a way to think about algebraic ideas.

This may suggest an idea for a workshop theme. I use Winplot regularly in my teaching here at IMSA. I could discuss its applications to various topics in the classroom. However, such a workshop would be probably be suitable for teachers of older students, perhaps Grades 8–12.