The Problem is the Solution:

Creating Original Problems in Gifted Mathematics Classes

**Overview of the Study**

Consider the following problem, composed (and subsequently solved) by a gifted high school sophomore for an assignment involving the writing of an original problem in a pre-calculus course:

The human eye is very unique, and it shapes our perception of the world around us. But why do we perceive things as we do, and how is this linked to mathematics? This is the question I set out to explore with this original problem. In everyday life, objects appear smaller, larger, thinner, thicker, and a host of other alterations depending on how we view them. I chose to study how the size of an object is affect[ed] by moving away or towards it. Looking online, I found a study to find the optimized angle, but I still wondered how mathematicians arrived at this point. The problem I am about to present is my attempt at blending reality and the world of mathematics to create a problem.

A man observes a painting hung on a wall from several feet away, looking directly at it. He realizes that the viewing angle changes as he moves away from the painting, getting larger until it hits a point ofoptimization, after which it begins getting smaller. However the man cannot accurately conclude that this supposed point exists where the angle is at its maximum without mathematics. Write a proof that proves the optimization to have a larger angle than all other possible points given the images below.

What kinds of instructional strategies effectively engage and sustain motivation among students who are gifted and talented in mathematics (or other domain)? Are students differently or more deeply engaged in mathematics when they are asked to create problems rather than answer them? Matsko (2011) presents a novel approach to and reflection on developing creative thinking skills and deepening conceptual understanding by asking gifted students in mathematics courses to create their own problems, like the “point of optimization problem” above:

Anyone can write tedious, difficult problems that review core math subjects, but to write problems in a novel, challenging, and refreshing manner, one must be imaginative. I feel that this creative side of math is an often overlooked aspect of the field as many believe math to be an extremely black-and-white, rigid, and boring subject.

In this study, we surveyed and examined the assignments of students enrolled in a specialized high school for students who are gifted and talented in mathematics and science. With Matsko’s (2011) experiences with students enrolled in Advanced Problem Solving (APS) in mind, we wondered:

1. whether the opportunity to create original mathematics problems would enhance levels of challenge, creativity, and intrinsic motivation and
2. whether first-semester sophomores in their problem-creation exercise would differ from upperclassmen in the same exercise.

Prior to developing this study, the exercise was piloted and refined with students in upper level mathematics courses such as APS and BC Calculus.

**Conceptual Framework**

Silver (1997), in examining multiple approaches to the instruction of mathematics, provides a departure point for this study:

Mathematics as an intellectual domain stands at or near the top of any hierarchical list of intellectual domains ordered according to the extent to which creativity is evident in disciplinary activity or production. Thus, it is ironic that for most students throughout the world, mathematics would almost certainly be among the set of school subjects least associated with creativity. (p.75)

Silver suggests that creativity is a disposition that is underdeveloped and underappreciated in mathematics instruction and that inquiry-based mathematics (including problem posing and problem solving) can effectively develop “deep, flexible knowledge in content domains.” (p.75) There is indeed a paucity of empirical literature that explores creativity and mathematics and specifically links motivation to creativity. Before exploring the literature in these areas, however, it is important to place this study in the conceptual framework of motivation theory.

Motivation in mathematics is of particular concern for educators in both gifted and general populations. There is evidence of general decline in motivation in mathematics from grade 9 through grade 11 (Chouinard and Roy, 2008). According Preckel, Goetz, Pekrun, and Kleine (2008), while, in the general population, mean differences in mathematics ability are relatively small, “males show higher mathematics competence beliefs, a stronger interest in math, and a stronger performance orientation in mathematics than do females.” (p. 149) With declines in motivation and gender-based motivational differences in mind, can math educators, as Silver (1997) suggests, enhance motivation through innovative learning opportunities?

Very broadly, this study is anchored in the concept of intrinsic motivation, which “refers to motivation to engage in an activity for its own sake” (Pintrich & Schunk 2002, p.245.) In other words, given an array of options to act upon, what do we choose to do because of our enjoyment of the activity?

Motivation may be understood as a process which begins with initial engagement and moves toward sustained engagement and self-regulated strategies. Therefore, this study is specifically characterized by dimensions of self-determination theory of motivation, as developed and explored by Deci and Ryan (2000, 2002.) Self-determination theory suggests that 1) individuals are aware of their needs, their strengths, and their weaknesses, and 2) decisions to act and satisfy needs depend on our understanding of them.

In particular, we believe that in the problem-creation exercises in this study, we will see evidence of two of the psychological needs/dimensions present in the decisions that direct behavior in self-determination theory, namely autonomy and competence. Feelings of competence are necessary because we must feel we have an ability to master both our environment and the multitude of interactions in our lives. Secondly, humans have a common and innate need to act autonomously and with a certain degree of control over their lives.

With respect to the mathematics problem-creation exercises explored in this study, we were interested in the ways in which and degrees to which students’ experiences enhance intrinsic motivation by:

1. Presenting a challenging activity, which enhances self-efficacy and feelings of competence;
2. Engaging curiosity about problems that seem complex or incongruous; and
3. Allowing students a sense of control and ownership over their own learning.

These dimensions are asserted by Lepper and Hodell (1989) as being integral to enhancing intrinsic motivation. A fourth dimension, fantasy, is not being explored in this study.

In the study of mathematics, Banda, Matuszny, and Therrien, (2009) suggest that engaging students in developing higher-order mathematics skills consists of presenting mathematics tasks that students prefer in order to enhance subsequent interest in solving difficult **math** tasks. To this point, learning in a non-traditional (i.e., non-teacher-centered) classroom has been shown to enhance student motivation and achievement. Ali, Akhter, Shahzad, Sultana, and Ramzan, (2011) suggest that a problem-based learning (PBL) experience in mathematics, because of its relevance and “real-world” design and approach, creates a sense of ownership of the content.

Problem-based learning requires that students arrive at a novel resolution to an ill-structured, real-life problem. In this study, however, we asked students to create rather than simply solve problems. We believe that the outcomes of problem-creation will be similar to those of PBL: enhanced motivation, retention, and conceptual understanding (Torp & Sage, 2002). Students have found the sequence of assignments that we will explore in this study valuable and have reported that they were able to think more conceptually as a result of doing them, and we intend to explore students’ attitudes more deeply.

Brunkalla (2009) has asserted a relationship between the development of creativity in mathematics and both conceptual understanding and motivation. Shriki (2010) suggests that allowing for original approaches to problem solving enhances both creativity and conceptual understanding.

The present study, however, is concerned with students who are gifted or high-achieving in mathematics. According to Sriraman (2005) the study of creativity in mathematics is a very small subset of research in the field of gifted education research. While this study does not isolate creativity *per se* as a variable, we are interested in determining whether creating mathematics problems enhances gifted students’ motivation and whether we see evidence of creativity in the type of problems they create.

**Methodology**

**Subjects**

Participants in this study comprised primarily sophomores and juniors enrolled in a three-year (sophomore, junior, and senior), residential high school for students identified as talented in mathematics and science. Because of the exploratory nature of this study, no comparison students were used.

Participants in the study were enrolled in Mathematical Investigations (MI). MI courses are required courses for students before moving on to either AB or BC Calculus course sequence. Table 1 presents the breakdown of participants by grade and gender.

Table 1: Participants by Gender, Grade

|  |  |  |  |
| --- | --- | --- | --- |
|  | Sophomore | Junior | Senior |
| Male | 36 | 13 | 0 |
| Female | 18 | 13 | 2 |
| Total | 54 | 26 | 2 |

One week prior to the administration of the instrument, the researcher and course instructor met with students in each designated class. Students were informed of the nature of the research project and the ways in which their responses would be used.

**Problem Creation**

Unlike most mathematics courses in which students are asked to complete a problem or set of problems to demonstrate mastery of mathematical concepts, the classes we investigated employed an assessment in which students were asked to generate original mathematics problems their area of interest. The creation of original problems is utilized in conjunction with typical homework assignments.

The problem-creation assignments consist of the writing of three original conceptual problems over the course of the semester. With the prompt that the problems be conceptual in nature (as opposed to routine problems which can be solved simply by applying a known solution method), students were encouraged to think in novel ways. For the first conceptual problem, there were some examples from a previous exam to give students an idea of what constitutes a “conceptual” problem.

The assignment itself consisted of four parts. The following steps were taken verbatim from the assignment sheet, and the full set of instructions for the assignment can be found in Appendix A

*1. Motivation: How did you come up with the problem? Was it based on a problem on the worksheets? An exam? A Problem Set? Were you doodling? Did it come to you in a dream? In the shower? Just a sentence or two will suffice here. But, importantly: acknowledge your source! It's OK to look at other problems, just cite them if you use them.*

*2. Problem Statement: Fairly self-explanatory. But a caution: give it to someone else to proofread! One of the most common traps to fall into is to write a problem which can be interpreted in more than one way. Is your problem stated absolutely clearly, so that someone else can understand it perfectly without needing to ask you any questions about interpretation?*

*3. Problem Solution: Again, self-explanatory. But your solution should be in paragraph form, using complete sentences! And if you only have a partial solution, you should explain where you are stuck and those questions whose answers could enable you to make further progress.*

*4. Reflection: Only a few sentences are necessary here. What did you learn? What did you observe about yourself as a problem writer? At the end of the semester, you will need to write an essay about your growth as a mathematician and problem-writer, so making notes along the way would be a good idea.*

After three of these assignments were completed, students were asked to write a brief reflective paper given the following prompts: (1) How did you grow as a problem-writer this semester? (2) Was this type of assignment valuable? Why or why not? This reflection was then submitted as part of a mini-portfolio, in which students compiled all their graded problems and their reflection in one pdf document.

**Data Gathering**

**Instrument.**

Students were surveyed in-class immediately following the submission of two of the three problem creation assignments. For this exploratory study, the instrument was a paper-and-pencil survey developed specifically to assess students’ engagement and motivation for the problem creation assignment. The survey presented Likert-type items that were derived from several sources: 1) trends in comments from students in prior classes in which problem creation was required of students; 2) course outcomes; 3) formative questions posed by the instructor following several years of similar class exercises.

The survey instrument consisted of 11 forced-choice and three open-ended questions (see Appendix B for the complete survey instrument). The open-ended responses provided an opportunity to gather information that is illustrated by supporting examples. Such open-ended questions permitted respondents to clarify what was of interest to them and allowed us to discern emerging issues and patterns even when there was no clear evidence requiring a research hypothesis or narrowly devised set of questions. The forced choice questions of the interview protocol, however, provided a vehicle to respond to *a priori* concerns or issues.

|  |
| --- |
| 1. Creating original mathematics problems helps me understand mathematics concepts more effectively than solving assigned problems.
 |
| 1. Creating original mathematics problems enhances my confidence in mathematics more generally.
 |
| 1. I am more engaged and interested in mathematics when I am allowed to create my own problems.
 |
| 1. Creating original mathematics problems is more satisfying than answering problems posed by the instructor.
 |
| 1. Creating original problems causes me to think about my own thinking (metacognition) more.
 |
| 1. The problems I choose to create I select because they are concepts I have difficulty with and want to understand better.
 |
| 1. The problems I choose to create I select because they are related to concepts I enjoy.
 |
| 1. The problems I choose to create I select because they are related to concepts I have not been able to explore enough in class.
 |
| 1. I find creating problems more challenging than answering problems posed by the instructor.
 |
| 1. The exercise of creating problems would have been valuable earlier on in my [high school] mathematics experience (e.g., during an MI course.)
 |
| 1. I find myself thinking about my original problems outside of class.
 |

The forced-choice questions were posed with a 5 point scale in which 1 = strongly disagree and 5 = strongly agree. The forced-choice items were followed by three open-ended follow-up questions:

* Has the exercise of creating original mathematics problems enhanced your motivation in math class? If so, in what ways?
* Has the exercise of creating original mathematics problems enhanced your ability to think creatively? If so, in what ways?
* Have you been able to transfer any of the skills you have developed in the creation of original problems to other courses, including courses outside of mathematics? Give specific examples, if possible.

**Data Analysis**

**Analysis of Likert-Scale Data**

Because we used a five-point Likert scale for the survey, between-groups analyses were conducted Mann-Whitney U test for significance. In the analysis by class, because there were only two seniors, we eliminated seniors and did not run analyses including the two female seniors.

In the first analysis, we asked whether there were gender-based differences on the individual items on the problem creation survey. In the first analysis, we asked whether there were differences between classes (juniors and seniors) on the same individual items. Results of the Mann-Whitney U test did not prove to be statistically significant in between-group differences on any of the eleven questions.

The fact that we did not find any gender-based differences may, in fact, be a notable finding. In the school in which the study was conducted, a significant amount of institutional research has been conducted in the area of gender attitudes toward mathematics and science. Indeed, a significant body of literature exists examining such concepts as self-efficacy, expectancy value, and course taking patterns in higher level mathematics courses. Our finding of no gender-based differences suggests that in future iterations of the problem creation exercise that we examine this question more deeply.

Although we treated the forced choice items as ordinal data for the purposes of between-groups analyses, we also examined the responses using descriptive statistics and calculated means in order to examine the relative rank of students’ responses to individual items. Interestingly, the item with a significantly higher mean than all other responses (m=3.95) was the agreement with the statement, “I find creating problems more challenging than answering problems posed by the instructor.” We found strong and varied support for this level of agreement in the open-ended responses. When we examined the ranking of the statements by mean, we found it interesting that three of the four highest ranked statements reflected challenge, enjoyment, and metacognition. Taken together, these ideas are consistent with the essential elements of optimal experience or flow, as defined by Csikszentmihalyi (1991.)

We were also particularly pleased with the students’ agreement with the statements that problem creation enhanced metacognition and that they seemed to derive their problems from concepts that they enjoy. Interestingly, students indicated that they generally did not create problems based on concepts with which they were having difficulty, the item that ranks lowest of all the forced choice items (m=2.71).

Table 2: Means of Responses to Problem Creation Survey

|  |  |  |  |
| --- | --- | --- | --- |
|  | n | m | sd |
| I find creating problems more challenging than answering problems posed by the instructor. | 86 | 3.95 | 1.187 |
| Creating original problems causes me to think about my own thinking (metacognition) more. | 86 | 3.51 | 1.244 |
| The problems I choose to create I select because they are related to concepts I enjoy. | 86 | 3.41 | 1.202 |
| Creating original mathematics problems helps me understand mathematics concepts more effectively than solving assigned problems. | 86 | 3.30 | 1.149 |
| I am more engaged and interested in mathematics when I am allowed to create my own problems. | 86 | 3.12 | 1.241 |
| I find myself thinking about my original problems outside of class. | 86 | 2.97 | 1.269 |
| The problems I choose to create I select because they are related to concepts I have not been able to explore enough in class. | 85 | 2.96 | 1.229 |
| Creating original mathematics problems is more satisfying than answering problems posed by the instructor. | 86 | 2.95 | 1.217 |
| Creating original math problems enhances my confidence in mathematics more generally. | 86 | 2.93 | 1.093 |
| The exercise of creating problems would have been valuable earlier on in my [high school] mathematics experience (e.g., during an MI course.) | 86 | 2.79 | 1.209 |
| The problems I choose to create I select because they are concepts I have difficulty with and want to understand better. | 86 | 2.71 | 1.136 |

**Analysis of Open-ended Items**

We were interested whether students’ autonomy in creating their own problems enhanced intrinsic motivation. Lepper and Hodell (1989) suggest that instructors can enhance autonomy by:

1. Presenting a challenging activity, which enhances self-efficacy and feelings of competence;
2. Engaging curiosity about problems that seem complex or incongruous; and
3. Allowing students a sense of control and ownership over their own learning.

In this section, we will analyze students’ responses to the survey’s open-ended items and attempt to identify ways in which their comments provide evidence of challenge, competence, curiosity, creativity, and control and ownership.

First, where do the problems originate? Prior to articulating problem and solution, each student is expected to describe briefly the source of motivation for the problem they created. In analyzing students’ responses to the question, we found evidence of the need for challenge, interdisciplinary thinking, transfer to real life and feelings of competence. Many of the motivation statements suggested that current or prior work in mathematics class was significant (e.g., “I came up with this problem by analyzing what we had done so far in math class and what more could be expanded upon.”) Others suggested that the questions originated with an idea or experience in a non-mathematics class (e.g., “My motivation for the problem came from SI physics. I just finished a unit on gravitation and found it particularly interesting.”) Over sixty percent of responses fell into these two categories.

Other responses, however, demonstrated the variety of sources from which students find ways to explore mathematics in novel, engaging ways. A number of students indicated that the motivation for their original problems was derived from interests or activities outside of school. One student found motivation in childhood interests:

I got my inspiration for this problem by remembering the times when I was younger and fascinated by airplanes and how their various wing shapes all gave them different flying traits. Obviously triangle shape was tied into that, and I thusly decided to write a problem on triangles so that my pent up nostalgia could be freely spent.

Another student found motivation for math akin to a personal pursuit:

What made this original problem fun was the fact that I couldn’t settle on what I wanted to make my problem about. So I thought about it for a while and realized that doing math reminds me of the joy I feel when dancing. With this I sought out a way to express my dancing excellence through math and unite these seemingly incompatible concepts.

One student created a problem that originated in school:

I was thinking about the flu and everything I’ve heard about how if one person gets sick, everyone in the school gets sick, so I decided to see how long it would take for that to actually happen [in my school].

Finally, another created a problem with an entrepreneurial bent:

I came up with this problem after reading a book on marketing strategies. While reading this book, I found out that phone users typically use their phones during a specific period of day. I also found that users who mostly talk with people from another country typically receive texts in intervals as the people from each region they talk to.

***No Recognition of Enhancement of Motivation.***

The follow-up surveys were equally revealing about students’ motivation. While the pre-experience survey helped the researchers investigate the source of their problem, the follow-up questions enabled the researchers the opportunity to investigate whether the problem-creation experience *per se* had any influence on their motivation in math class. Many of the students indicated that the problem creation exercise had no discernible impact on the motivation in mathematics class. Their reasons for stating so, however, varied. “The problems have given an overwhelming amount of stress.” “I am motivated in math without problem creation.” “They do not necessarily relate to what we are learning in class.”

Our first response to such responses was to suggest that students found no motivation in the exercise. Each of these reasons was cited multiple times to the question of effect of the assignment on motivation. Because the survey question was open-ended and because it did not provide any prompts, responses such as these suggest that a focus group protocol or an interview session might, in fact, allow students to identify ways that the exercise might be valuable (e.g., the student who created the problem integrating mathematics and dance.)

It is also probable that students interpreted the term “motivation” in multiple ways. We approached the study with our own operationalized constructs, and we discerned evidence of such constructs. The above comments, however, might suggest that students may not recognize that motivation can relate to feelings of stress, for example. Thus, to ask students to self-asses their motivation may lead us to consider new or re-worded with prompts that were derived from responses in this exploratory study.

***Evidence of Challenge Enhancing Motivation.***

Specifically with respect to Lepper and Hodell’s (1989) conditions for enhancing intrinsic motivation (presenting a challenge, engaging curiosity about problems that seem complex or incongruous, and allowing a sense of control over learning), we found ample evidence of student engagement leading to an enhanced sense of efficacy, deeper engagement, and feelings of ownership and intrinsic motivation.

In several responses, we found that the challenge of problem creation leading to enhanced intrinsic motivation. “It has caused me to explore intriguing math problems more intensely.” “It has gotten me to work harder.” “I tend to be more focused and give more attention to a problem I am interested in learning.”

We also found that students used the exercise to challenge themselves beyond the concepts presented in the course. “I explore and self-study concepts that I do not know.” “I have the need to understand the basics as quick as possible so that I can go on to more complex things.” “I begin with an original problem related to what we learned and then go above and beyond.” “Creating original math problems has made me think beyond what is in the classroom and then making me feel better about myself.”

We also recognized in several responses that students had, indeed used the opportunity to take ownership for their learning, and, in one case, a shift from performance orientation (grades). “It is much more interesting figuring stuff out on your own instead of being guided.” “I am able to explore topics that interest me so I can care more about the solution. I have more motivation to solve the problem rather than just getting a good grade.

***Evidence of Curiosity Enhancing Motivation.***

Throughout this article, we have suggested that motivation is a process that begins with initial engagement. Thus framed, we found several instances of students who found the problem creation exercise an opportunity to engage with math in a new and novel way. “I pick fascinating and interesting topics that allow me to get motivated to solve that problem if it is the last thing I do.” “When I come across a problem I have difficulty with, I think about similar problems I could make using that concept.”

***Other Findings.***

One of the clear trends discerned from the open-ended questions was the degree to which students indicated that the assignment enhanced their metacognition. Interestingly, there was no prompt or probe in the open-ended questions asking specifically about metacognition, and the responses that supported this trend came primarily from the question of how the exercise enhanced their creativity. “It has made me more aware of my problem solving skills.” “It has helped with my questioning skills.” “Some of my original problems were related to chemistry and physics, so I could apply it to other fields of study.” “I think it has taught me to think differently and more open-mindedly about the place of math in everyday life.”

We also asked students whether or not the assignment enhanced their ability to transfer the concepts and skills to another setting. While it is difficult to fully assess students’ actual ability to transfer the skills through a self-report survey, we were intrigued by the ways in which the skills developed through problem creation were manifested in students’ responses. “I used my original problem to find out population logistics of snowy owl populations.” “In classes that have a lot of discussion, it is now easier for me to think of my own original solutions.” “I find myself creating problems using things from everyday life. My last problem mixed Pokemon, physics, and calculus in one problem.”

**Discussion and Recommendations**

If we think of motivation as a process that begins with initial engagement and moves toward self-directed, self-regulated learning, then the value of this study has implications beyond mathematics and, indeed, beyond gifted and talented learners. Students enrolled in the school under investigation in this study are admitted on the basis of demonstrated talent and interest in mathematics and science. Further, all students are required to be enrolled in mathematics. Thus, with this specialized population in mind, this study pointed us to several clear trends which support the research literature on motivation and inform classroom practice.

First, when given the freedom to design their own learning experiences, students will make connections to events, experiences, and memories of deep significance to them. We saw evidence here not only of motivational constructs such as engagement, self-regulation, and autonomy, but also of optimal experience or flow as characterized by Csikszentmhalyi (1991), such as with the student who found “that doing math reminds me of the joy I feel when dancing.”

Second, although we were exploring motivation and creativity in math, we found that, without asking them to articulate it as such, students were able to suggest how the problem-creation exercise enhanced the development of their metacognitive strategies.

**Areas for Future Research**

This study examined the relationship between mathematics, creativity, and motivation. Given both the design of this study and the findings posed in this paper, we believe that there are rich possibilities for inquiry in a variety of areas.

First, our study focused on a novel approach to teaching mathematics. Delimiting the study to mathematics poses a number of research possibilities across domains, grades, and skill levels. What would be the student response to problem creation in disciplines other than mathematics? There are obvious parallels to the sciences (particularly physics.) Similarly, how might such an approach be received by middle school students? Also, would the pedagogical approach be as effective with students who are not identified as gifted in mathematics?

Second, this study used self-report measures developed from several sources. We recognize this as a legitimate but limited way to assess creativity, motivation, and metacognition for our study. Such a limitation, however, leads to several research possibilities. First, would the use of the problem creation activity enhance these traits as measured by standardized instruments? Second, are the effects of enhancing these traits evident in subsequent math classes?

Third, we did not include measures of student achievement as a variable. The students’ responses provided evidence of enhanced awareness of metacognitive skills, but would the students demonstrate improvement on indicators of math achievement (such as math grades, in-class test performance, or AP performance)?

Finally, our non-statistically significant finding from the forced-choice items poses areas for deeper inquiry into gender and novel approaches to teaching and learning in mathematics. For example, does such an exercise mitigate differences in the value which students ascribe to mathematics?

**Implications for the Classroom**

The self-directed nature of problem selection is critical to the success of the assignments. Some students write and solve problems at a level far beyond that of the course material. It would be impossible to present such students a similar challenge by giving the entire class the same homework assignment. The instructor reports that he occasionally gains new insights by reading problem solutions, as some students tackle traditional topics with fresh perspectives.

How might a mathematics teacher use such an assignment in his or her classroom? Tailoring the assignment to the teacher’s background is important. Having taught at the university level for fourteen years, the instructor in this course is comfortable with a more open-ended approach, but for less-experienced teachers, the idea of having students write “conceptual” problems may be daunting. A narrower focus could still prove beneficial. For example, “Write a word problem whose solution involves the Pythagorean Theorem,” or “Create a problem involving rate, time, and distance which involves Sam riding a unicycle at a rate of 3m/s.” In such cases, the “Motivation” section of the assignment may not be necessary.

Assigning grades is highly individual in nature. For this assignment, the instructor used a flexible letter-grade rubric where an A means the problem is conceptual and well-written, a B means the problem is too routine or too many errors are present in the solution, and a C means a lack of effort or a clear last-minute attempt at doing the assignment. Using “+” and “-” allows for further distinction. Thus, if a student genuinely tries, he or she will not earn a low grade, but also will not earn an A unless the work is well-done. Because of the wide range of assignments students turn in, a detailed rubric is not particularly helpful. Moreover, instructors need some flexibility. The instructor gave the example of a pre-calculus student who tackled writing a calculus problem – although the solution had quite a few errors, the student still earned an A- as a result of a genuine and relatively successful effort to go far beyond the course material.

Teachers must also decide upon the level of detail for writing comments on problems. For more capable students, comments on details of mathematical style are appropriate, while for students who have difficulty solving the problem they pose, more general comments are given.

Finally, as with many types of instruction with which teachers do not have adequate experience, problem creation requires planning and a level of comfort in assessing conceptual understanding. Given what our students generated and the sources of their motivation, we were both surprised and very pleased with the thoughtfulness and creativity demonstrated in their original work. And, as you demonstrate your enthusiasm for the assignment, you may generate similar excitement for creating problems among your students.

**Limitations of the Present Study**

This study originated with the researchers’ interests in mathematics pedagogy and student motivation and creativity. While we found rich, thoughtful responses from the students, and while we were pleased with the results of this exploratory, we acknowledge several limitations to this study.

What we did not ask in this study was how students valued and were motivated in mathematics relative to other subjects. Therefore, we do not have a measure of relative value of mathematics for each student, which may have influenced their responses on the survey.

Methodologically, this study had several inherent weaknesses. First, while this was an exploratory study, the subjects were all enrolled in classes taught by the same instructor, and we could not discern whether the effects we found could be attributed to the teacher’s style or personality. The study also would have been strengthened by instrumentation that would allow comparison to normative data.

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Appendix A:

Student Instructions for Original Problem Creation

Your next task is to write an original problem! Specifically, you should come up with one good problem suitable for the conceptual part of an exam (like questions 7 and 8 from the last exam). You could write a problem based on logarithms (because you are already familiar with that material, and this makes problem writing a bit easier.) Or base your problem on a particularly interesting problem from a recent Problem Set. Or polynomials! It's up to you. But here is the important idea: try to come up with a problem which involves more than just going through several steps in a routine manner.

So how should you approach writing problems? Notice that the conceptual problems on the first exam were based on ideas on the worksheets we did in class { but problems which were not just “follow the steps," but which had a little more to them. There is no “right" way to do this (just try something! Experiment, draw, doodle, meditate, dance, scream, juggle, defenestrate, whatever.) Discuss it with a friend.

So here's what you need to include when you write an original problem:

1. Motivation: How did you come up with the problem? Was it based on a problem on the worksheets? An exam? A Problem Set? Were you doodling? Did it come to you in a dream? In the shower? Just a sentence or two will suffice here. But, importantly: acknowledge your source! It's OK to look at other problems, just cite them if you use them.

2. Problem Statement: Fairly self-explanatory. But a caution: give it to someone else to proofread! One of the most common traps to fall into is to write a problem which can be interpreted in more than one way. Is your problem stated absolutely clearly, so that someone else can understand it perfectly without needing to ask you any questions about interpretation?

3. Problem Solution: Again, self-explanatory. But your solution should be in paragraph form, using complete sentences! And if you only have a partial solution, you should explain where you are stuck and those questions whose answers could enable you to make further progress.

4. Reflection: Only a few sentences are necessary here. What did you learn? What did you observe about yourself as a problem writer? At the end of the semester, you will need to write an essay about your growth as a mathematician and problem-writer, so making notes along the way would be a good idea.

**Appendix B**

The purpose of this survey is to assess students’ interest, engagement, and feelings of competence in the area of creating original mathematics problem. The results of this survey will inform both Dr. Matsko’s instruction as well as the development of new courses and assessments in IMSA’s math courses. Taking this survey is entirely voluntary. No identifying information is necessary.

Grade □ Sophomore □Junior □ Senior

Gender □ Male □ Female

*Please indicate your level of agreement with the following statements by placing an X in the appropriate column to the right of each item.*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **STRONGLY****AGREE** |  | **AGREE** |  | **STRONGLY DISAGREE** |
| 1. Creating original mathematics problems helps me understand mathematics concepts more effectively than solving assigned problems.
 | □5 | □4 | □3 | □2 | □1 |
| 1. Creating original mathematics problems enhances my confidence in mathematics more generally.
 | □5 | □4 | □3 | □2 | □1 |
| 1. I am more engaged and interested in mathematics when I am allowed to create my own problems.
 | □5 | □4 | □3 | □2 | □1 |
| 1. Creating original mathematics problems is more satisfying than answering problems posed by the instructor.
 | □5 | □4 | □3 | □2 | □1 |
| 1. Creating original problems causes me to think about my own thinking (metacognition) more.
 | □5 | □4 | □3 | □2 | □1 |
| 1. The problems I choose to create I select because they are concepts I have difficulty with and want to understand better.
 | □5 | □4 | □3 | □2 | □1 |
| 1. The problems I choose to create I select because they are related to concepts I enjoy.
 | □5 | □4 | □3 | □2 | □1 |
| 1. The problems I choose to create I select because they are related to concepts I have not been able to explore enough in class.
 | □5 | □4 | □3 | □2 | □1 |
| 1. I find creating problems more challenging than answering problems posed by the instructor.
 | □5 | □4 | □3 | □2 | □1 |
| 1. The exercise of creating problems would have been valuable earlier on in my IMSA mathematics experience (e.g., during an MI course.)
 | □5 | □4 | □3 | □2 | □1 |
| 1. I find myself thinking about my original problems outside of class.
 | □5 | □4 | □3 | □2 | □1 |
|  |  |  |  |  |  |

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Has the exercise of creating original mathematics problems enhanced your motivation in math class?

If so, in what ways?

Has the exercise of creating original mathematics problems enhanced your ability to think creatively?

If so, in what ways?

Have you been able to transfer any of the skills you have developed in the creation of original problems to other courses, including courses outside of mathematics? Give specific examples, if possible.